Automatica 50 (2014) 1143–1150

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A performance oriented multi-loop constrained adaptive robust tracking control of one-degree-of-freedom mechanical systems: Theory and experiments ☆

Lu Lu, Bin Yao [1]

The State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, China
School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA

ARTICLE INFO

Article history:
Received 26 September 2011
Received in revised form
24 August 2013
Accepted 25 December 2013
Available online 28 February 2014

Keywords:
Motion control
Constrained control
Input saturation
Adaptive control
Linear motors

ABSTRACT

A performance oriented multi-loop approach to the adaptive robust tracking control of one-degree-of-freedom mechanical systems with input saturation, state constraints, parametric uncertainties and input disturbances is presented. The control system contains three loops. In the outer loop, constrained optimization algorithms are developed to generate a replanned trajectory on-line at a low sampling rate so that the converging speed of the overall system response to the desired target is maximized while not causing input saturation and the violation of state constraints. In the inner loop, a constrained adaptive robust control (ARC) law is synthesized and implemented at high sampling rate to achieve the required robust tracking performances with respect to the replanned trajectory even with various types of uncertainties and input saturation. In the middle loop, a set-membership identification (SMI) algorithm is implemented to obtain a tighter estimate of the upper bound of the inertia so that more aggressive replanned trajectory could be used to further improve the overall system response speed. Interaction of the three loops is explicitly characterized by a set of inequalities that the design variables of each loop have to satisfy. It is theoretically shown that the resulting closed-loop system can track feasible desired trajectories with a guaranteed converging time and steady-state tracking accuracy without violating the state constraints. Experiments have been carried out on a linear motor driven industrial positioning system to compare the proposed multi-loop constrained ARC algorithm with some of the traditional control algorithms. Comparative experimental results obtained confirm the superior performance of the proposed algorithm over existing ones.

1. Introduction

Nowadays, performance requirements for motion control are becoming increasingly stringent for industrial applications. Both the steady-state tracking accuracy requirement and certain transient performances (such as the fast response speed) have to be met. On the other hand, the mechanical systems to be controlled are often subject to various types of physical constraints such as the input saturation and state constraints. The systems may also experience large extent of parametric uncertainties, uncertain nonlinearities and disturbances. Traditionally, high gain feedback based approaches are often used to address the disturbance rejection issue in motion control problems so that good steady-state tracking accuracy can be achieved. To deal with parametric uncertainties, parameter adaptation laws can be added to the feedback structure (Astrom & Wittenmark, 1994). Various types of control strategies have been developed along this line of thought process (Krstic, Kanellakopoulos, & Kokotovic, 1995; Polycarpou & Ioannou, 1993). Adaptive robust control (ARC) developed in the past two decades is a good example of this kind of control strategies to deal with disturbances and parametric uncertainties (Hong & Yao, 2007a; Yao & Tomizuka, 1996) with a number of successful applications. However, this type of feedback control strategies cannot handle physical constraints of the systems very well. It is difficult to maximize the converging speed of the overall

☆ The work is supported in part by the National Basic Research and Development Program of China under 973 Program Grant 2013CB035400 and the Science Fund for Creative Research Groups of National Natural Science Foundation of China (No: 51221004). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Kyung-Soo Kim under the direction of Editor Toshiharu Sugie.

E-mail addresses: lulu.lvlv@gmail.com (L. Lu), byao@ieee.org (B. Yao).

[1] A Chang Jiang Chair Professor at Zhejiang University and a Professor at Purdue University (on leave). Tel.: +1 765 494 7746; fax: +1 765 494 0539.

http://dx.doi.org/10.1016/j.automatica.2014.02.003
0005-1098/

system response to the desired trajectory under different physical constraints when the initial tracking errors are large.

Contrary to the high gain feedback based approaches, optimal control is a class of control strategy that could handle the constraints of the system and achieve certain optimal transient performance (e.g. minimum converging time) in a systematic way (Mayne, Rawlings, Rao, & Scokaert, 2000; Sage & White, 1968). However, it is well known that when strong disturbances and uncertainties are present, the performance of the optimal control might be very poor. Since no high gain feedback is incorporated in the optimal control law, the steady-state tracking error could be significantly large, and even the closed-loop stability is questionable. Partly because of these, they are seldom used in the precision motion control of mechanical systems.

The above quick review of literature reveals that the problem of maintaining closed-loop stability while attempting to track a non-conservative desired trajectory with small steady-state tracking error and fast converging speed under physical constraints (e.g., the control input saturation) has not been well addressed. The aim of this paper is to develop a performance oriented multi-loop constrained adaptive robust control approach to solve the problem stated above for one-degree-of-freedom (1-DOF) mechanical systems. The main idea of the proposed approach is to implement an on-line constrained time-optimal trajectory planning algorithm in the outer-loop in conjunction with the conventional ARC controller in the inner-loop such that not only the good steady-state tracking accuracy of the conventional ARC approach can be preserved, but also the converging speed of the tracking error to the targeted steady-state tracking accuracy can be maximized under physical constraints. In addition, a middle loop is also included to obtain tighter estimation bounds of the inertia using the set-membership identification technique (Fogel & Huang, 1982) so that more aggressive on-line trajectory replanning can be used to further improve the overall system converging speed. Although more complicated than the traditional algorithms, it is theoretically and experimentally demonstrated that the closed-loop system with the proposed algorithm is able to simultaneously achieve good steady-state tracking performance and fast transient response speed, which could not be achieved with any of the existing algorithms.

The rest of the paper is organized as follows: Section 2 presents the system dynamics and formulates the problem to be solved; Sections 4 to 6 present the inner-loop, middle-loop and outer-loop designs, respectively. Section 7 formulates the overall control law. Section 8 presents the comparative experimental results on a 1-DOF linear motor driven stage. Section 9 concludes the paper.

2. Problem formulation

The dynamics of 1-DOF mechanical system can be represented by the following equation:

$$\begin{aligned}&\dot{x}_1 = x_2,\\ &M\dot{x}_2 = S(u) + \boldsymbol{\varphi}^T(x_1, x_2, t)\boldsymbol{\theta} + \Delta(x_1, x_2, t),\\ &y = x_1\end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1,\ x_2]^T$ is the state of the system that is measurable. $y = x_1$ represents the position of the system to be controlled, with its velocity denoted as x_2. M represents the unknown inertia of the system normalized with respect to the electrical gain of the control input. $\boldsymbol{\theta} \in R^m$ is the vector of unknown parameters of the system, and $\boldsymbol{\varphi}(x_1, x_2, t) \in R^m$ is the vector of known regressors corresponding to $\boldsymbol{\theta}$. The term $\boldsymbol{\varphi}^T(x_1, x_2, t)\boldsymbol{\theta}$ may incorporate some nonlinearities existing in the mechanical system such as the nonlinear frictions and the electro-magnetic forces. $u \in R$ is the control input to the system, and the nonlinear function $S(u)$ representing the input saturation constraint of the system is given by

$$S(u) = \begin{cases} u, & \text{if } |u| \le u_M \\ u_M \operatorname{sign}(u), & \text{if } |u| > u_M, \end{cases} \tag{2}$$

where u_M is the input saturation limit. $\Delta(x_1, x_2, t)$ represents the lumped model uncertainties including the input disturbances. The system is also subject to the following state constraints:

$$\mathbf{x} \in \Omega_x \overset{\Delta}{=} \left\{[x_1\ x_2]^T : x_{1min} \le x_1 \le x_{1max},\ x_{2min} \le x_2 \le x_{2max}\right\}, \tag{3}$$

in which x_{1min}, x_{1max} are the constant position bounds, and $x_{2min} < 0, x_{2max} > 0$ are the constant velocity bounds. All these bounds are known in practice.

The following assumptions are made for the unknown parameters $M, \boldsymbol{\theta}$ and the uncertainty term $\Delta(x_1, x_2, t)$:

Assumption 1. The extent of the parametric uncertainties and uncertain disturbances are known, i.e.,

$$\begin{aligned}&M \in \Omega_M \overset{\Delta}{=} \{M : M_{min} \le M \le M_{max}\}\\ &\boldsymbol{\theta} \in \Omega_{\boldsymbol{\theta}} \overset{\Delta}{=} \{\boldsymbol{\theta} : \boldsymbol{\theta}_{min} \le \boldsymbol{\theta} \le \boldsymbol{\theta}_{max}\}\\ &\delta_l(x_1, x_2, t) \le \Delta(x_1, x_2, t) \le \delta_u(x_1, x_2, t),\\ &|\delta_l(x_1, x_2, t)| \le d, \qquad |\delta_u(x_1, x_2, t)| \le d,\\ &\quad \forall x_1 \in [x_{1min}, x_{1max}], x_2 \in [x_{2min}, x_{2max}],\ t \ge 0,\end{aligned} \tag{4}$$

where $M_{min} > 0$, $\boldsymbol{\theta}_{min} = [\theta_{1min}, \ldots, \theta_{(m)min}]^T$, $\boldsymbol{\theta}_{max} = [\theta_{1max}, \ldots, \theta_{(m)max}]^T$. $\delta_l(x_1, x_2, t)$ and $\delta_u(x_1, x_2, t)$ are the known bounding functions for $\Delta(x_1, x_2, t)$, and d is a known constant value denoting the upper bound for their absolute values.

Assumption 2.

$$\begin{aligned}&\phi_l(x_1, x_2, t) \le \boldsymbol{\varphi}^T(x_1, x_2, t)\boldsymbol{\theta} \le \phi_u(x_1, x_2, t),\\ &|\phi_l(x_1, x_2, t)| \le h, \qquad |\phi_u(x_1, x_2, t)| \le h\\ &\quad \forall x_1 \in [x_{1min}, x_{1max}], x_2 \in [x_{2min}, x_{2max}], \boldsymbol{\theta} \in \Omega_{\boldsymbol{\theta}},\end{aligned} \tag{5}$$

where $\phi_l(x_1, x_2, t)$ and $\phi_u(x_1, x_2, t)$ are the known bounding functions for $\boldsymbol{\varphi}^T(x_1, x_2, t)\boldsymbol{\theta}$, and h is a known constant value denoting the upper bound of their absolute values. (For simplicity, ϕ_l and ϕ_u are assumed to have the same known constant upper bound.)

Assumption 3.

$$h + d < u_M. \tag{6}$$

The output trajectory tracking problem is considered in this paper. The objective is to design a control law $u(t)$ such that, in the presence of input saturation and disturbances, the output $y(t)$ converges to the desired output $y_d(t)$ as fast as possible and the tracking error $e_{yd}(t) = y(t) - y_d(t)$ at the steady state is as small as possible. The desired output $y_d(t)$ is assumed to be second-order differentiable with the following assumption:

Assumption 4.

$$\begin{aligned}&x_{1min} < x_{1dmin} \le y_d(t) \le x_{1dmax} < x_{1max},\\ &x_{2min} < x_{2dmin} \le \dot{y}_d(t) \le x_{2dmax} < x_{2max},\\ &|\ddot{y}_d(t)| \le \ddot{y}_{dmax} < \frac{u_M - h - d}{M_{max}}.\end{aligned} \tag{7}$$

3. Control structure

In this section, a novel hybrid control structure shown in Fig. 1 is proposed to solve the problem in a holistic way such that all