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Brief paper

Distributed receding horizon control of large-scale nonlinear systems: Handling communication delays and disturbances[☆]

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ABSTRACT

This paper studies the robust distributed receding horizon control (DRHC) problem for large-scale continuous-time nonlinear systems subject to communication delays and external disturbances. A dual-mode robust DRHC strategy is designed to deal with the communication delays and the external disturbances simultaneously. The feasibility of the proposed DRHC and the stability of the closed-loop system are analyzed, and the sufficient conditions for ensuring the feasibility and stability are developed, respectively. We show that: (1) the feasibility is affected by the bounds of external disturbances, the sampling period and the bound of communication delays; (2) the stability is related to the bounds of external disturbances, the sampling period, the bound of communication delays and the minimum eigenvalues of the cooperation matrices; (3) the closed-loop system is stabilized into a robust invariant set under the proposed conditions. A simulation study is conducted to verify the theoretical results.

1. Introduction

The distributed receding horizon control (DRHC), also known as the distributed model predictive control, has found many practical applications in large-scale systems such as multi-vehicle systems (Dunbar & Murray, 2006; Izadi, Gordon, & Zhang, 2009; Keviczky, Borrelli, & Balas, 2006; Keviczky, Borrelli, Fregene, Godbole, & Balas, 2008), power systems (Venkat, Hiskens, Rawlings, & Wright, 2008) and chemical processes (Liu, Chen, de la Peña, & Christofides, 2012). For these distributed large-scale systems, the centralized receding horizon control (RHC) is computationally expensive to be implemented due to the large number of dimensions. In contrast, the DRHC which treats large-scale systems as many interconnected subsystems based on communication networks, is efficient in reducing the computational complexity, and has received considerable attention during last few years.

In the literature on DRHC, the research can be generally classified into two types according to different system structures. The first type of research is focused on large-scale systems in which subsystem dynamics are coupled. Such large-scale systems are artificially partitioned into many coupled subsystems, and the DRHC including a communication strategy among subsystems is required to be designed for stabilizing the overall system. Some promising results along this line, for example, can be referred to Camponogara, Jia, Krogh, and Talukdar (2002), Dunbar (2007), Stewart, Wright, and Rawlings (2011) and Venkat et al. (2008). In the second type of research, the large-scale system consists of many physically decoupled subsystems but with a coupled control objective function. Each subsystem is capable of communicating with several subsystems, and an RHC needs to be designed for each subsystem such that, the overall system is stabilized and the control objective is achieved. This paper studies the DRHC problem along the line in the second type of research.

In the past few years, great attention has been paid to investigating the DRHC problem in the second type of research, and many results have been reported for studying large-scale linear systems, nonlinear systems and their applications. For large-scale linear systems, a DRHC strategy has been designed to address coupled constraints and external disturbances in Richards and How (2007); Franco et al. have studied the DRHC problem by considering delayed state exchange in Franco, Parisini, and Polycarpou (2007);

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the distributed LQR problem has been investigated for subsystems with identical dynamics in Borrelli and Keviczky (2008); the linear DRHC scheme with communication delays has been applied for tracking control of multi-vehicle systems in Izadi et al. (2009). For large-scale nonlinear systems, Dunbar and Murray have designed the DRHC strategy for multiple vehicles with continuous-time dynamics in Dunbar and Murray (2006), and Keviczky et al. have investigated the discrete-time counterpart in Keviczky et al. (2006); in Franco, Magni, Parisini, Polycarpou, and Raimondo (2008), the work (Franco, Parisini, & Polycarpou, 2007) has been extended for nonlinear systems; in Keviczky et al. (2008), a nonlinear DRHC strategy has been designed for coordination and formation control of multiple autonomous vehicles. The robust DRHC that deals with the continuous-time nonlinear systems subject to bounded disturbances has been proposed in our recent work in Li and Shi (in press), where the relationship between the bounds of disturbances and the system stability has been exploited. To explicitly address the communication delays, the delay-involved DRHC strategy has been designed in Li and Shi (2013a).

It is well recognized that the design of the DRHC strategy for practical large-scale systems is apt to encounter two main issues simultaneously. On the one hand, the communication networks deployed among subsystems tend to be unreliable, resulting in communication delays (Li & Shi, 2012, 2013b). This issue has been exclusively studied, for example, in Izadi et al. (2009) and Li and Shi (2013a). On the other hand, the disturbances (due to environmental noises or unmodeled system dynamics) are almost unavoidable from the perspective of practical implementation, which has inspired the work in Franco et al. (2008), Franco, Parisini, and Polycarpou (2007), Li and Shi (in press) and Richards and How (2007). However, most of the existing results are focused only on one issue while ignoring the other. To the best of the authors' knowledge, the design of the DRHC strategy for nonlinear systems by simultaneously considering the communication delays and disturbances has not been investigated, and the co-effects of these two issues to the DRHC are unknown, which motivates this study.

In this paper, we further research by considering a more practical case: large-scale nonlinear systems simultaneously subject to bounded disturbances and communication delays. Extensions to jointly consider communication delays and external disturbances in DRHC design bring essential technical difficulties including how to formulate the new optimization problem to design the DRHC algorithm and how to analyze the feasibility and stability, which are resolved in this paper. The main contributions of this study are three-fold.

- A dual-mode robust DRHC strategy, being capable of handling the communication delays and external disturbances simultaneously, is proposed. In order to address the co-effects of the communication delays and external disturbances, the robustness constraint (Li & Shi, 2013a, in press) is incorporated into the optimization problem. In addition, a post-predicted state trajectory (which is defined in (8)) is generated when communication delays occur, and the post-prediction state is designed as the initial system state for the optimization Problem 1.
- The feasibility of the proposed robust DRHC is analyzed and the feasible conditions are developed. We show that the iterative feasibility of the DRHC is related to the bounds of the disturbances, the sampling period and the upper bound of the communication delays, given the initial feasibility. The specific bounds of these parameters on ensuring the feasibility, are proposed.
- The robust stability of the closed-loop large-scale system is established and the sufficient conditions on ensuring the robust stability are developed. We show that, the stability of the closed-loop system is affected by the upper bounds of the disturbances, the sampling period, the upper bound of the communication delays and the minimum eigenvalues of the cooperation matrices. Under the developed conditions, the large-scale system can be stabilized into a robust invariant set.

The notations used in the paper are as follows. The symbol $\mathbb{N}$ represents the set of all positive integers; the symbol $\mathcal{M}$ is defined as the collection $\{1, 2, \ldots, M\}$, $M \in \mathbb{N}$, and $\mathbb{R}^n$ stands for the n-dimensional real space. The superscript "T" and "-1" denote the transpose and the inverse operation of a matrix, respectively. Given a matrix P, $P > 0$ ($P \geqslant 0$) means the matrix is positive definite (positive semi-definite). The 2-norm of a given column vector v is denoted by $\|v\|$ and the P-weighted norm is defined as $\|v\|_P \triangleq \sqrt{v^T P v}$, where P is a given matrix with appropriate dimension. Given two matrices Q_1 and Q_2, $\bar{\lambda}(Q_1)$ and $\underline{\lambda}(Q_1)$ represent the maximum and minimum of the absolute values of the eigenvalues for the matrix Q_1; the notation $\lambda(Q_1, Q_2)$ is defined as $\lambda(Q_1, Q_2) \triangleq \underline{\lambda}(Q_1)/\bar{\lambda}(Q_2)$, where $\bar{\lambda}(Q_2) \neq 0$. We use the notation $x(t_i; t_j)$ to represent the value of $x(t_i)$ produced at time t_j, where $t_i \geqslant t_j \geqslant 0$, and $x(t_j; t_j) = x(t_j)$.

2. Problem statement and preliminaries

Consider the DRHC problem for a large-scale nonlinear system in which each subsystem (agent) $\mathcal{A}_i$, $i \in \mathcal{M}$, is described as:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \omega_i(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the system state, $u_i(t) \in \mathbb{R}^m$ is the control input, $\omega_i(t) \in \mathbb{R}^n$ is the external disturbance. For each agent $\mathcal{A}_i$, the control input is constrained as $u_i(t) \in \mathcal{U}_i \subset \mathbb{R}^m$, where $\mathcal{U}_i$ is a compact set which contains the origin as an interior point. $\omega_i(t)$ belongs to a compact set $\mathcal{W}_i$ and is bounded by $\rho_i \triangleq \sup_{\omega_i(t) \in \mathcal{W}_i} \|\omega_i(t)\|$. The nominal system of the system in (1) can be represented as:

$$\dot{\bar{x}}_i(t) = f_i(\bar{x}_i(t), u_i(t)). \tag{2}$$

By defining $x(t) = \mathrm{col}(x_1(t), \ldots, x_M(t))$, $u(t) = \mathrm{col}(u_1(t), \ldots, u_M(t))$, $\omega(t) = \mathrm{col}(\omega_1(t), \ldots, \omega_M(t))$, $f(x, u) = \mathrm{col}(f_1(x_1, u_1(t)), \ldots, f_M(x_M, u_M))$, $\mathcal{U} = \mathcal{U}_1 \times \cdots \times \mathcal{U}_M$ and $\mathcal{W} = \mathcal{W}_1 \times \cdots \times \mathcal{W}_M$, the large-scale nonlinear system without network interaction and communication delays can be represented as:

$$\dot{x}(t) = f(x(t), u(t)) + \omega(t), \tag{3}$$

where $u(t) \in \mathcal{U}$ and $\omega(t) \in \mathcal{W}$.

In the large-scale system, each agent $\mathcal{A}_i$ is able to communicate with some agents according to their physical distances. The neighbors of the agent $\mathcal{A}_i$ are defined as the agents from which it can receive information. The set of the indices for the neighbors of agent $\mathcal{A}_i$ is denoted as $\mathcal{N}_i$ and the concatenated state vector of neighbors of the agent $\mathcal{A}_i$ is denoted as $x_{-i}(t)$. There are time delays of the transmitted information among the communication links deployed in each agent and its neighbors. The system states of the large-scale system in (3) are coupled in the objective (or cost) function. The objective of this paper is to design a robust DRHC scheme for the large-scale system in (3) subject to communication delays and external disturbances, such that the overall system is robustly stabilized.

Before proceeding, some standard assumptions associated with the system dynamics are made. The same assumptions have been made in Dunbar (2007), Dunbar and Murray (2006) and Li and Shi (2013a, in press).

Assumption 1. For each agent $\mathcal{A}_i$, $i \in \mathcal{M}$, with the system dynamics in (1), assume that: (a) $f_i(0, 0) = 0$ and $f_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a