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Brief paper

Self-triggered linear quadratic control[☆]

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ABSTRACT

Self-triggered control is a recently proposed paradigm that abandons the more traditional periodic time-triggered execution of control tasks with the objective of reducing the utilization of communication resources, while still guaranteeing desirable closed-loop behavior. In this paper, we introduce a self-triggered strategy based on performance levels described by a quadratic discounted cost. The classical LQR problem can be recovered as an important special case of the proposed self-triggered strategy. The self-triggered strategy proposed in this paper possesses three important features. Firstly, the control laws and triggering mechanisms are synthesized so that a priori chosen performance levels are guaranteed by design. Secondly, they realize significant reductions in the usage of communication resources. Thirdly, we address the co-design problem of jointly designing the feedback law and the triggering condition. By means of a numerical example, we show the effectiveness of the presented strategy. In particular, for the self-triggered LQR strategy, we show quantitatively that the proposed scheme can outperform conventional periodic time-triggered solutions.

1. Introduction

In many control applications, controllers are nowadays implemented using communication networks in which the control task has to share the communication resources with other tasks. Despite the fact that resources can be scarce, controllers are typically still implemented in a time-triggered fashion, in which control tasks are executed periodically. This design choice often leads to over-utilization of the available communication resources, and/or causes a limited lifetime of battery-powered devices, as it might not be necessary to execute the control task every period to guarantee the desired closed-loop performance. Also in the area of 'sparse control' (Gallieri & Maciejowski, 2012), in which it is desirable to limit the changes in certain actuator signals while still realizing specific control objectives, periodic execution of control tasks may not be optimal either. In both networked control systems with scarce communication resources and sparse control applications arises the *fundamental* problem of determining optimal sampling and communication strategies, where optimality needs to reflect both implementation cost (related to the number of communications and/or actuator changes) as well as control performance. It is expected that the solution to this problem results in control strategies that abandon the periodic time-triggered control paradigm.

Two approaches that abandon the periodic communication pattern are event-triggered control (ETC), see, e.g., Arzén (1999), Åström and Bernhardsson (1999) and Donkers and Heemels (2012), Heemels, Sandee, and van den Bosch (2008), Heemels et al. (1999), Henningsson, Johannesson, and Cervin (2008), Lunze and Lehmann (2010), Tabuada (2007) and Wang and Lemmon (2009), and self-triggered control (STC), see, e.g., Almeida, Silvestre, and Pascoal (2010, 2011), Anta and Tabuada (2010), Donkers, Tabuada, and Heemels (2012), Mazo, Anta, and Tabuada (2010), Velasco, Fuertes, and Marti (2003) and Wang and Lemmon (2009). Although ETC is effective in the reduction of communication or actuator movements, it was originally proposed for different reasons, including the reduction of the use of computational resources

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and dealing with the event-based nature of the plants to be controlled. In ETC and STC, the control law consists of two elements being a feedback controller that computes the control input, and a triggering mechanism that determines when the control input has to be updated. The difference between ETC and STC is that in the former the triggering consists of verifying a specific condition continuously and when it becomes true, the control task is triggered, while in the latter at an update time the next update time is pre-computed. ETC laws have been mostly developed for continuous-time systems, although they have also appeared for discrete-time systems, see, e.g., Cogill (2009), Eqtami, Dimarogonas, and Kyriakopoulos (2010), Heemels and Donkers (2013), Li and Lemmon (2011), Yook, Tilbury, and Soparkar (2002), Molin and Hirche (2013) and Lehmann (2011, Sec. 4.5). In addition, in Arzén (1999), Henningsson et al. (2008), Heemels et al. (2008) and Heemels, Donkers, and Teel (2013) so-called periodic event-triggered control strategies were proposed and analyzed for continuous-time systems.

At present ETC and STC form popular research areas. However, two important issues have only received marginal attention: (i) the co-design of both the feedback law and the triggering mechanism, and (ii) the provision of performance guarantees (by design). To elaborate on (i), note that current design methods for ETC and STC are mostly emulation-based approaches, by which we mean that the feedback controller is designed without considering the scarcity in the system's resources. The triggering mechanism is only designed in a subsequent phase, where the controller has already been fixed. Since the feedback controller is designed before the triggering mechanism, it is difficult, if not impossible, to obtain an optimal design of the combined feedback controller and triggering mechanism in the sense that the minimum number of control executions is achieved while guaranteeing closed-loop stability and a desired level of performance.

Regarding (ii), only a few available ETC/STC methods provide quantitative analysis tools for control performance, such as $\mathcal{L}_2$-gains, quadratic cost, $\mathcal{H}_2$ type of criteria, and so on. For instance, in Donkers and Heemels (2012) one can analyze the ETC/STC loop a posteriori and evaluate what the $\mathcal{L}_\infty$-gain is, and clearly by doing this for various choices of the triggering mechanism one can (indirectly) synthesize a controller with a good closed-loop $\mathcal{L}_\infty$-gain (in balance with a reasonable communication usage) based on an iterative design process. A similar procedure can be applied for the $\mathcal{L}_2$-gain, see, e.g., Wang and Lemmon (2009). Alternatively, using Lunze and Lehmann (2010) and Yook et al. (2002), one can tune the parameters of the event-triggering condition (once the controller is fixed) to obtain a desirable ultimate bound on the state. In addition, a few ETC and STC methods exist that aim at minimizing a criterion involving besides control cost also communication cost (Cogill, 2009; Li & Lemmon, 2011; Molin & Hirche, 2013). However, in most cases they do not provide guarantees in terms of standard (LQR, $\mathcal{L}_2$, $\mathcal{H}_2$) control cost (i.e., without the presence of communication cost), and, in fact, due to the resulting absolute threshold in the event-triggering mechanism, these control cost are typically not finite. The case of continuous-time linear systems with a quadratic performance measure (LQR) is studied in Velasco et al. (2011) and Yepez, Velasco, Marti, Martin, and Fuertes (2011). Both papers aim at arriving at ETC laws that yield the *same* cost as the optimal LQR controller, but require less communication than the continuously updating optimal LQR controller. The main idea behind the approach is to maximize the time until the next control value update, considering that the rest of the (future) controller executions will be according to standard periodic time-triggered updates. In Velasco et al. (2011), the controller design is emulation-based, whereas Yepez et al. (2011) studies a co-design method for both the feedback law and the triggering condition. However, no formal guarantees are given in these papers about the true cost realized by the ETC implementation, and the framework in Velasco et al. (2011) and Yepez et al. (2011) does not offer a possibility to "trade" performance for less communication resource usage.

The main contribution of the present paper is a novel STC strategy for discrete-time linear systems in the presence of stochastic disturbances, addressing the issues (i) and (ii) and allowing to trade guaranteed performance levels with utilization of communication resources. The contribution of this paper is threefold:

- the methods guarantee a desired performance level based on quadratic (discounted) cost without an iterative design process. In fact, the presented strategy aims at reducing the use of communication resources, while still guaranteeing a prespecified sub-optimal level of performance;
- for the considered control problem, we solve a co-design problem by jointly designing the feedback controller and the triggering mechanism;
- by means of a numerical example, we demonstrate quantitatively that aperiodic control can outperform periodic control when both control performance and communication cost are important.

1.1. Nomenclature

Let $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and the set of non-negative integers, respectively. The notation $\mathbb{N}_{\geq s}$ and $\mathbb{N}_{[s,t)}$ is used to denote the sets $\{r \in \mathbb{N} \mid r \geq s\}$ and $\{r \in \mathbb{N} \mid s \leq r < t\}$, respectively, for some $s, t \in \mathbb{N}$. The inequalities $\prec$, $\preceq$, $\succ$ and $\succeq$ are used for matrices, i.e., for a square matrix $X \in \mathbb{R}^{n\times n}$ we write $X \prec 0$, $X \preceq 0$, $X \succ 0$ and $X \succeq 0$ if X is symmetric and, in addition, X is negative definite, negative semi-definite, positive definite and positive semi-definite, respectively. Sequences of vectors are indicated by bold letters, e.g., $\boldsymbol{u} = (u_0, u_1, \ldots, u_M)$ with $u_i \in \mathbb{R}^{n_u}$, $i \in \{0, 1, \ldots, M\}$, where $M \in \mathbb{N} \cup \{\infty\}$ will be clear from the context. Let X and Y be random variables. The expected value of X is denoted by $\mathbb{E}(X)$ and the conditional expectation of X given Y is denoted $\mathbb{E}[X \mid Y]$. The trace of a matrix A is denoted by $\mathrm{tr}(A)$.

2. Self-triggered linear quadratic control

In this section, we provide the problem formulation and the general setup for the self-triggered control strategy. We consider the control of a discrete-time LTI system given by

$$x_{t+1} = Ax_t + Bu_t + Ew_t, \tag{1}$$

for $t \in \mathbb{N}$, where $x_t \in \mathbb{R}^{n_x}$ is the state, $u_t \in \mathbb{R}^{n_u}$ is the input and $w_t \in \mathbb{R}^{n_w}$ is the disturbance, respectively, at discrete time $t \in \mathbb{N}$. We assume that the pair (A, B) is controllable and that w_t, $t \in \mathbb{N}$, are independent and identically distributed random vectors (not necessarily Gaussian distributed) with $\mathbb{E}[w_t] = 0$ and $\mathbb{E}[w_t w_t^\top] = I$, $t \in \mathbb{N}$, where $I \in \mathbb{R}^{n_w\times n_w}$ is the identity matrix. In this section, we are interested in control strategies that guarantee a certain control performance in terms of a discounted quadratic cost function

$$J = \sum_{t=0}^{\infty} \mathbb{E}\left[\alpha^t \left(x_t^\top Q x_t + 2x_t^\top S u_t + u_t^\top R u_t\right) \mid x_0\right], \tag{2}$$

involving the weighting matrices Q, R and S, where $\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \succ 0$. The discount factor $0 < \alpha \leq 1$ is assumed to be strictly less than one when $E \neq 0$ to assure that (2) is bounded. Note that $E = 0$ and $\alpha = 1$ allow us to consider an LQR-like framework. If we assume that the state is available at every $t \in \mathbb{N}$ and also that the control input can be updated at every $t \in \mathbb{N}$, it is well known