Automatica 50 (2014) 1296–1303

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

# Robust tracking controller design for non-Gaussian singular uncertainty stochastic distribution systems☆

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ARTICLE INFO

*Article history:*
Received 24 January 2013
Received in revised form
10 September 2013
Accepted 8 February 2014
Available online 13 March 2014

*Keywords:*
Singular systems
Output stochastic distribution control
Robust tracking control
Augmentation control
Linear matrix inequality (LMI)

ABSTRACT

In this paper, the robust tracking control problem for uncertain singular stochastic distribution control (SDC) systems is considered. A new control target, where the distribution tracking error at each time instant satisfies a certain upper bound beyond a limited time, is proposed. This control target is different from the tracking control in the output SDC systems which makes the output probability density function (PDF) track a desired PDF as close as possible. Then an instant performance index instead of the infinite integration index is adopted, and the upper bound of this index is taken as the stability condition of a Lyapunov function to obtain a robust tracking controller via an augmentation control and linear matrix inequality (LMI). Simulations are also included to show the effectiveness of the proposed algorithm and encouraging results have been obtained.



## 1. Introduction

Stochastic control, as one of important areas in control engineering practice, has drawn a considerable attention in the past decades. Many approaches have been developed and used in practical control systems. Examples are minimum variance control (Astrom, 1970), linear quadratic Gaussian (LQG) (Anderson & Moore, 1989) and Markov stochastic control (Stroock, 2009). These methods are all based on the assumption that the systems and the variables subject to Gaussian distribution. However, this assumption is not always true in some practical systems such as the length distribution of fiber in the paper industry and the shape distribution of the flames in boilers (Wang, 2000). In these cases, variance is not sufficient to characterize the statistical properties of random output. Motivated by this problem, some new approaches called stochastic distribution control (SDC), such as the output SDC (Wang, 2000), the system SDC or fully probabilistic control design (Herzallah & Karny, 2011; Karny, 1996; Karny & Kroupa, 2012) and the process SDC (Annunziato & Borzi, 2013; Forbes & Forbes, 2004; Pigeon, Perrier, & Srinivasan, 2011; Zhu & Zhu, 2011; Zhu, Zhu, & Yang, 2012), were proposed which directly control the probability density function (PDF) of the concerned. Those types of stochastic control systems are much more general than the traditional ones and can therefore be applied to either Gaussian or non-Gaussian dynamic systems. The output SDC method, as one of the major methods, has exhibited a rapid development and a number of interesting control algorithms including minimum entropy control, optimal tracking control, and robust tracking control have been developed (Chen & Wang, 2007; Guo & Wang, 2010; Guo & Yin, 2009; Wang, 1999; Wang & Wang, 2004; Wang & Zhang, 2001; Zhou & Wang, 2005).

It is noted that some developed PDF control laws were obtained directly by using numeral optimization calculation (Wang, 2000; Wang & Wang, 2004; Wang & Zhang, 2001), where the analysis of the closed-loop properties including stability and robustness is difficult to perform because a fixed closed-loop structure in the numeral optimization processes rarely exists. Therefore, it is necessary to develop output SDC control strategies that have a fixed closed-loop structure (Chen & Wang, 2007; Guo & Wang, 2010; Guo & Yin, 2009; Wang, 1999; Zhou, Yue, & Wang, 2005). Among those algorithms, an optimal tracking approach was developed by using optimal control theory for the deterministic output SDC system in Zhou et al. (2005) so as to realize the perfect tracking control based on the square root *B*-spline SDC

☆ This work was supported by NSFC (Grant Nos. 61004045, 61333007, 61290323, and 61174128), and Chinese Fundamental Research Funds for Central Universities (ZZ1013). Those supports are gratefully acknowledged. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Valery Ugrinovskii under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2014.02.032
0005-1098/

model. As is well known, in practice it is difficult to obtain an accurate model. Thus, the robustness is an important issue for all dynamic systems to be considered. For the output SDC systems, a robust tracking control strategy has been proposed via LMI for the output SDC system with random parameters (Chen & Wang, 2007). Subsequently, Guo and his collaborators have also formulated some robust tracking control algorithms via the convex optimization approach (Guo & Wang, 2010; Guo & Yin, 2009). In those mentioned output SDC systems, most of the control targets are to make the output PDF track a desire PDF as close as possible. Obviously, two PDFs complete overlap situation is the closest. However, in practice, because of disturbances and the sampling, there is scarcely two complete coincidence PDFs. In other words, we need a definition of what kind of tracking is considered as the as close as possible tracking. The most readily accepted is that one of the tracking performance evaluation indices of two PDFs, such as the squared difference and the moments, does not exceed a threshold. This forms one purpose of our work in this paper that the robust controller is designed with a new control target, with which the distribution tracking error in each moment satisfies a certain upper bound beyond a limited time. The most closely related work is that of Guo and Wang (2010) where a peak-to-peak performance index is applied to construct a robust tracking controller by LMI and the multiple control objectives including stabilization, tracking performances, robustness and state constraints can be guaranteed simultaneously. However, in this paper, not only the modeling uncertainty and disturbance, but also approximation errors are considered as the ideal approximation does not exist in practice.

As described above, the robust tracking control has been widely used in the output SDC systems design. However, it is worth pointing out that hitherto, almost all of those studies are focused on the nonsingular output SDC systems and to a certain degree, the singular systems have more convenient and natural description for practical systems (Xu & Lam, 2004). Indeed, the studies of singular system are mainly focused on three approaches. They are state space method, geometry method and polynomial matrix. Most singular system stability problems are solved by state space approaches for the reason that state space methods are maturely studied for nonsingular systems (Jun, 2012; Lu, Du, & Xue, 2008; Ma & Boukas, 2008, 2009; Wang, Xue, & Lu, 2008; Wang & Zhang, 2012; Xu, Ji, & Chu, 2006; Xu & Lam, 2004). Lu and his fellows devoted a lot on the singular system control (Lu et al., 2008; Wang et al., 2008). Other researchers also bring the singular system into either filter or fault diagnose area (Lee & Fong, 2007; Yao, Cocquempot, & Wang, 2010; Yao, Qin, Wang, & Jiang, 2012). Although the singular system problem is widely discussed and a normal output SDC system can be transformed into a singular output SDC system (Wang, 2000), only a few work has been reported in the singular output SDC systems. This forms another purpose of the work in this paper where the robust tracking control algorithm of non-singular output SDC systems is extended to the singular systems by an instant performance index description. This idea can be also found in Yao et al. (2010, 2012) where the main purpose is designed a fault diagnosis and detection (FDD) and fault tolerant control (FTC) algorithms in a deterministic singular system and the controller is still a traditional tracking output SDC controller with numeral optimization.

The rest of this paper is organized as follows. The problem description is given in Section 2. In Section 3, a distribution error robust tracking algorithm is proposed. Simulation results are included in Section 4, which is followed by some concluding remarks in Section 5.

## 2. Problem description

In the dynamic stochastic systems, define $\upsilon(t) \in [a, b]$ as a uniformly bounded random variable to describe the output of the system and $u(t) \in R^{q\times 1}$ is the input vector to control the distribution shape of the output. At any time, the distribution shape of $\upsilon(t)$ can be represented by its PDF $\gamma(y, u(t))$ (Wang, 2000),

$$P\{a \le \upsilon(t) \le \zeta, u(t)\} = \int_a^\zeta \gamma(y, u(t))dy \tag{1}$$

where $P\{a \le \upsilon(t) \le \zeta, u(t)\}$ presents the probability of the random variable $\upsilon(t)$ inside $[a, \zeta]$ with control $u(t)$. In practice, it is difficult to obtain the exact expression of the PDF albeit the distribution sample can be obtained easily. Therefore, an approximation expression can be obtained using the following $B$-spline approximation (Wang, 2000),

$$\sqrt{\gamma(y, u(t))} = \sum_{i=1}^{n} w_i(u(t))B_i(y) + e_0(y, t) \tag{2}$$

where $B_i(y)$, $(i = 1, 2, \ldots n)$ are pre-specified basis function defined on the interval $y \in [a, b]$, $w_i(u(t))$ $(i = 1, 2, \ldots n)$ are the weights which are controlled by the inputs $u(t)$, and $e_0(y, t)$ is the approximation error, $\int_a^b e_0^T(y, t)e_0(y, t)dy \le \delta_0$ and $\delta_0$ is a small positive number. The PDF satisfies the following condition,

$$\int_a^b \gamma(y, u(t))dy = 1. \tag{3}$$

This means that only $n - 1$ weights are independent. According to some simple transformation, the Eq. (2) can be rewritten as Wang (2000),

$$\sqrt{\gamma(y, u(t))} = C(y)V(t) + h(V(t))B_n(y) + e_0(y, t) \tag{4}$$

where $C(y) = [B_1(y), B_2(y), \ldots, B_{n-1}(y)]$, $V(t) = [w_1(u(t)), w_2(u(t)), \ldots, w_{n-1}(u(t))]^T$ is the weight vector. Denoted $\Sigma_0 = \int_a^b C^T(y)C(y)dy$, $\Sigma_1 = \int_a^b C^T(y)B_n(y)dy$, $\Sigma_2 = \int_a^b B_n^2(y)dy$, if $e_0(y, t) = 0$, we have

$$h(V(t)) = \frac{-\Sigma_1 V(t) \pm \sqrt{V^T(t)\left(\Sigma_1^T\Sigma_1 - \Sigma_2\Sigma_0\right)V(t)}}{\Sigma_2}.$$

Due to the continuous, $h(V(t))$ is a nonlinear function satisfying the following Lipschitz condition

$$\|h(V_1) - h(V_2)\| \le \|M(V_1 - V_2)\| \tag{5}$$

for any $V_1(t)$ and $V_2(t)$, where $M$ is a known matrix.

Taking the model uncertainty and disturbance into account, assuming $V(t)$ and $u(t)$ satisfy the following dynamic singular stochastic distribution control systems,

$$\begin{aligned} E\dot{V}(t) &= A(t)V(t) + B(t)u(t) \\ \sqrt{\gamma(y, u(t))} &= C(y)V(t) + h(V(t))B_n(y) + e_0(y, t) \end{aligned} \tag{6}$$

where $E$ is a singular matrix, $\gamma(y, u(t))$ is the output PDF and $e_0(y, t)$, $\int_a^b e_0^T(y, t)e_0(y, t)dy \le \delta_t$ is an approximation distribution errors where its nature can be estimated by statistical methods (Hazewinkel, 1987). $A(t)$ and $B(t)$ are time-varying parameters which can be written as follows Chen and Wang (2007),

$$A(t) = A + \Delta A, \qquad B(t) = B + \Delta B. \tag{7}$$

$A$ and $B$ are proper constant matrices. $\Delta A$ and $\Delta B$ are the modeling uncertainties which can be described by the following equation.

$$[\Delta A, \Delta B] = HF(t)[E_1, E_2]$$

where $H$, $E_1$ and $E_2$ are proper constant matrices. $F(t)$ is an variable matrix and satisfies $F^T(t)F(t) \le I$.