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Brief paper

Notes on sliding mode control of two-level quantum systems[☆]

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ABSTRACT

In [Dong, D. and Petersen, I.R. (2012). Sliding mode control of two-level quantum systems. Automatica, 48, 725–735], a sliding mode control approach has been proposed for two-level quantum systems to deal with bounded uncertainties in the system Hamiltonian. This paper further extends these results in two directions. One extension is to consider the effect of uncertainties during the process of driving the system's state back to the sliding mode domain from outside and we propose two approaches to accomplish this control task. The other extension generalizes the previous design approach to consider uncertainties described as perturbations in the free Hamiltonian.

The developing control theory for quantum systems is becoming an active research area (D'Alessandro, 2007; Dong & Petersen, 2010; Rabitz, de Vivie-Riedle, Motzkus, & Kompa, 2000). Several useful tools from classical control theory including optimal control theory, learning control and feedback control have been introduced to control system analysis and design for quantum systems (Altafini, 2007; Bolognani & Ticozzi, 2010; Boscain & Mason, 2006; Chia & Wiseman, 2011; D'Alessandro & Dahleh, 2001; Doherty & Jacobs, 1999; Khaneja, Brockett, & Glaser, 2001; Nurdin, James, & Petersen, 2009; Rabitz et al., 2000; Ticozzi & Viola, 2009; Wiseman & Milburn, 2010; Zhang & James, 2011; Zhang, Wu, Li, & Tarn, 2010). In particular, there exist many types of uncertainties (including noise, disturbance, decoherence, etc.) for most practical quantum systems and the robust control problem for quantum systems has been recognized as a key issue in developing practical quantum technology (Doherty et al., 2000; Dong, Lam, & Petersen, 2010; Pravia et al., 2003; Yamamoto & Bouten, 2009; Yi, Cui, Wu, & Oh, 2011; Zhang & Rabitz, 1994). Several methods have been proposed for the robust control of quantum systems. For example, James, Nurdin, and Petersen (2007) have formulated and solved a quantum robust control problem using the H^∞ method for linear quantum stochastic systems. D'Helon and James (2006) used the small gain theorem to analyze the robustness of a quantum feedback network. Li and coworkers (Li, 2011; Li & Khaneja, 2009) presented an ensemble control method which could be looked on as an open-loop robust control approach for quantum systems. In Dong and Petersen (2009, 2012), we developed a sliding mode control approach to enhance the robustness of quantum systems with uncertainties in the system Hamiltonian. In particular, two approaches based on sliding mode design have been proposed for the control of quantum systems in Dong and Petersen (2009) and potential applications of sliding mode control to quantum information processing have been presented. Dong and Petersen (2012) presents a detailed sliding mode control method for two-level quantum systems to deal with bounded uncertainties in the system Hamiltonian. This paper focuses on the sliding mode control of two-level quantum systems and extends the results in Dong and Petersen (2012) to deal with additional types of uncertainties (Dong & Petersen, 2011).

Sliding mode control generally includes two main steps: selecting a sliding surface (sliding mode) and controlling the system's state to and maintaining it in a sliding mode domain. We select an eigenstate $|0\rangle$ of the free Hamiltonian of a two-level quantum system as a sliding mode. Being in the sliding mode guarantees that the quantum system has the desired dynamics. Furthermore, we also define a sliding mode domain $\mathscr{D}$ in which the system's state has a small probability to collapse out of $\mathscr{D}$ when making a measurement. Then, such a sliding mode control problem includes two important subtasks: (I) design a control law to maintain the system's state in $\mathscr{D}$; and (II) design a control law to drive the system's state back to $\mathscr{D}$ if a measurement operation takes it away from $\mathscr{D}$. In Dong and Petersen (2012),

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we assumed that there exist no uncertainties during the control phase (II) and proposed a Lyapunov-based approach to accomplish such a subtask (Altafini, 2007; Kuang & Cong, 2008; Mirrahimi, Rouchon, & Turinici, 2005; Wang & Schirmer, 2010). This paper first proposes two controller design approaches (dependent on different situations) to accomplish subtask (II) where the uncertainties are not ignored. For subtask (I), assuming that there are no uncertainties which are described as perturbations in the free Hamiltonian, (Dong & Petersen, 2012) presented a periodic measurement based method to guarantee the desired robustness and also gave an approach to design the measurement period. In this paper, we give modified measurement periods which need to be used when we consider uncertainties described as perturbations in the free Hamiltonian.

This paper is organized as follows. Section 1 introduces the sliding mode control problem for two-level quantum systems. In Section 2, we present a controller design method to accomplish subtask (II) taking into account uncertainties. Section 3 presents modified measurement periods when we consider uncertainties described as perturbations in the free Hamiltonian. Concluding remarks are given in Section 4.

1. Sliding mode control of two-level quantum systems

The quantum control model under consideration can be described as (we have assumed $\hbar = 1$ by using atomic units)

$$\begin{aligned} &i|\dot{\psi}(t)\rangle = (H_0 + H_\Delta + H_u)|\psi(t)\rangle, \\ &|\psi(t=0)\rangle = |\psi_0\rangle, \end{aligned} \tag{1}$$

where the quantum state $|\psi(t)\rangle$ corresponds to a two-dimensional unit complex vector in a Hilbert space, the free Hamiltonian $H_0 = \frac{1}{2}\sigma_z$, the uncertainties $H_\Delta = \delta(t)I_z + \varepsilon_x(t)I_x + \varepsilon_y(t)I_y$ ($\delta(t)$, $\varepsilon_x(t)$, $\varepsilon_y(t) \in \mathbf{R}$), the control Hamiltonian $H_u = \sum_{k=x,y,z} u_k(t)I_k$, ($u_k(t) \in \mathbf{R}$, $I_k = \frac{1}{2}\sigma_k$) and the Pauli matrices $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ take the following form:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

Furthermore, we assume that the uncertainties are bounded:

$$|\delta(t)| \le \delta \quad (\delta \ge 0), \qquad \sqrt{\varepsilon_x^2(t) + \varepsilon_y^2(t)} \le \varepsilon \quad (\varepsilon > 0).$$

In practical applications, we often use the density operator ρ to describe the quantum state of a quantum system. For a two-level quantum system, the state ρ can be represented in terms of the Bloch vector $\mathbf{r} = (x, y, z) = (\mathrm{tr}\{\rho\sigma_x\}, \mathrm{tr}\{\rho\sigma_y\}, \mathrm{tr}\{\rho\sigma_z\})$:

$$\rho = \frac{1}{2}(I + \mathbf{r} \cdot \sigma). \tag{3}$$

The dynamical equation of ρ can be written as

$$\dot{\rho} = -i[H, \rho] \tag{4}$$

where $[A, B] = AB - BA$. After we represent the state ρ with the Bloch vector, the pure states (satisfying $\rho = |\psi\rangle\langle\psi|$) of a two-level quantum system correspond to the surface of the Bloch sphere, where $(x, y, z) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\theta \in [0, \pi]$, $\varphi \in [0, 2\pi]$. An arbitrary pure state $|\psi\rangle$ for a two-level quantum system can be represented as

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle, \tag{5}$$

where $|0\rangle$ and $|1\rangle$ are eigenstates of H_0.

To deal with the uncertainties H_Δ, Dong and Petersen (2012) have proposed a sliding mode control approach where the sliding mode S is defined as a functional of the state $|\psi\rangle$ and the Hamiltonian H; i.e., $S(|\psi\rangle, H) = 0$. In particular, the eigenstate $|0\rangle$ has been identified as the sliding mode. We further define a sliding mode domain $\mathscr{D} = \{|\psi\rangle : |\langle 0|\psi\rangle|^2 \ge 1 - p_0, 0 < p_0 < 1\}$, where p_0 is a given constant (Dong & Petersen, 2012). The definition of the sliding mode domain implies that the system's state has a probability of at most p_0 (which we call the probability of failure) to collapse out of $\mathscr{D}$ when making a measurement with the measurement operator σ_z. We expect to drive and then maintain a two-level quantum system's state in a sliding mode domain $\mathscr{D}$. However, the uncertainties H_Δ may take the system's state away from $\mathscr{D}$. Since a measurement operation unavoidably makes the measured system's state collapse, we expect that the control laws designed will guarantee that the system's state remains in $\mathscr{D}$ except that a measurement operation may take it away from $\mathscr{D}$ with a small probability (not greater than p_0).

In Dong and Petersen (2012), we divide the control task under consideration into three main subtasks. Since we can make a measurement on any initial state to drive it into $|0\rangle$ or $|1\rangle$, we only consider the following two important subtasks: (I) design a control law to maintain the system's state in $\mathscr{D}$; and (II) design a control law to drive the system's state back to $\mathscr{D}$ if a measurement operation takes it away from $\mathscr{D}$. Ref. Dong and Petersen (2012) ignores the uncertainties H_Δ during control process (II) and the uncertainties $\delta(t)I_z$ (i.e., there we assumed $\delta(t) \equiv 0$) for subtask (I). This paper will relax these two assumptions. First, we propose two controller design approaches for subtask (II) with uncertainties. To simplify the problem of controller design, we employ the measurement periods in Dong and Petersen (2012). Then we present modified measurement periods to guarantee the desired robustness for subtask (I) when $\delta(t)I_z$ ($|\delta(t)| \le \delta$) exists.

2. Control design for subtask (II) with uncertainties

In Dong and Petersen (2012), we have proposed a Lyapunov-based design approach to drive the quantum system's state back to the sliding mode domain when we ignore the uncertainties during the control process. This section will consider the case where the uncertainties are not ignored and propose a simple method to accomplish subtask (II). For an arbitrary initial state, we first make a projective measurement to drive it to $|0\rangle$ or $|1\rangle$. The general control algorithm used in this section can be described as follows: (i) select an eigenstate $|0\rangle$ of H_0 as a sliding mode $S(|\psi\rangle, H) = 0$, and define the sliding mode domain as $\mathscr{D} = \{|\psi\rangle : |\langle 0|\psi\rangle|^2 \ge 1 - p_0, 0 < p_0 < 1\}$. (ii) For the initial state $|1\rangle$, design a control law to drive the system's state into $\mathscr{D}$ using information on ε. (iii) For given p_0 and ε, design the period T for periodic projective measurements. (iv) For an initial state, make a projective measurement, then repeat the following operations. If the result is $|0\rangle$, make periodic projective measurements with a period T to maintain the system's state in $\mathscr{D}$. If the measurement result corresponds to $|1\rangle$, use the corresponding control law to drive the state back into $\mathscr{D}$.

In the control algorithm, the design of a control law in (ii) and the measurement period T in (iii) are two important tasks. We use the results of Dong and Petersen (2012) to design the measurement period T. Hence, this section focuses on the design of a control law in (ii). In the following, we consider two situations $H_\Delta = \varepsilon(t)I_\xi$ ($\xi = x$ or y) and $H_\Delta = \varepsilon(t)(\sin\varphi I_x - \cos\varphi I_y)$ (where $\varphi \in [0, 2\pi]$ is a constant). $H_\Delta = \varepsilon(t)I_\xi$ is a special case of $H_\Delta = \varepsilon(t)(\sin\varphi I_x - \cos\varphi I_y)$. The effect of a noisy environment on many practical systems in superconducting nanocircuits and solid-state nuclear magnetic resonance can be approximated as an uncertainty in the form of these uncertainties (e.g., see Möttönen, de Sousa, Zhang, & Whaley, 2006, Paladino, Faoro, Falci, & Fazio, 2002, Pravia et al., 2003). Also, the systematic errors in quantum operations considered in Brown, Harrow, and Chuang (2004) are a special case of $H_\Delta = \varepsilon(t)(\sin\varphi I_x - \cos\varphi I_y)$.