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Survey paper

The early days of geometric nonlinear control[☆]

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ABSTRACT

Around 1970 the study of nonlinear control systems took a sharp turn. In part, this was driven by the hope for a more inclusive theory which would be applicable to various newly emerging aerospace problems lying outside the scope of linear theory, and also by the gradual realization that tools from differential geometry, and Lie theory in particular, could be seen as providing a remarkably nice fit with what seemed to be needed for the wholesale extension of linear control theory into a nonlinear setting. This paper discusses an initial phase of the development of geometric nonlinear control, including material on the broader context from which it emerged. We limit our account to developments occurring up to the early 1980s, not because the field stopped developing at that point but rather to limit the scope of the project to something manageable. Even so, because of the volume and diversity of the literature we have had to be selective, even within the given time frame.

1. Introduction

This paper discusses an initial phase of the development of geometric nonlinear control, including material on the broader context from which it emerged. We limit our account to developments occurring up to the early 1980s, not because the field stopped developing at that point but rather to limit the scope of the project to something manageable. Even so, because of the volume and diversity of the literature we have had to be selective, in some cases only skimming the surface. The many applications, ranging from momentum wheel control of satellites and the generation of robotic trajectories, to the acrobatics of falling cats, provide effective advertisement for the relevance of the subject and some of these applications are discussed here as well. With the goal of reaching an audience wider than just those involved in this area of research, and mindful of the fact that some of the concepts involved are not everyday fare for engineers, we include considerable background material to enable suitably motivated readers not working in the area to better appreciate the what and why that lies behind the who and when.

Much of the early work in this area was related to applications in fields as disparate as satellite control, path planning for mobile robots and the design of excitation sequences for magnetic resonance spectroscopy. As was the case when state space methods were being introduced to describe linear systems, sometimes new methods are dismissed as being too theoretical, however in the case of differential geometric control this attitude seems to reflect an unfamiliarity with the mathematics being used rather than any

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lack of applicability. In fact, without attempting to mount an argument involving ever changing, and methodologically questionable, citation statistics and impact factors, suffice it to say that when control science is viewed in a wider context, the additional scope these ideas give to the theory and practice is impressive. Section 12 of this survey sketches a few applications that lie quite outside the reach of linear theory. In general, we give more prominence here to those parts of the theory that relate to applied problems.

2. Differential equations and transfer functions

In the 1960s the development of state space methods recast the way students of control learned about linear systems, changing the subject from one based on Laplace transforms and transfer functions to one in which vector spaces and first order linear differential equations took the center of the stage. At that time linear algebra was not part of the standard curriculum for engineering students (Matlab did not exist!) and this state space revolution had the effect of making linear algebraic ideas part of the everyday vocabulary of control engineers. While this was beneficial in that it opened up additional points of contact with mathematics and physics, it had the side effect of putting the field of control somewhat apart from previously neighboring engineering disciplines. It even created something of a schism within the field itself, as some declined to learn the new language. However, driven by concomitant developments in optimization theory, notably the maximum principle of Pontryagin, Boltyanskii, Gamkrelidze, and Mishchenko (1959), numerical methods for trajectory optimization (Kelley, Kopp, & Moyer, 1966), the work of Fisk (1963), Itô (1946) and Stratonovich (1963) on stochastic differential equations, the Kalman–Bucy filter Kalman and Bucy (1961), etc., and aided by the availability of well written expositions such as Kalman's papers on linear systems (Kalman, 1960a, 1963) and some excellent text books on linear algebra (e.g. Gantmacher (1959) and Halmos (1958)) this "linear revolution" proceeded quickly, if not painlessly.[1] The main ideas underpinning linear system theory have a clear mathematical structure, and key concepts such as controllability and feedback invariants, provided a rough template for how a more comprehensive nonlinear theory might develop.

In the middle of the 19th century, the work of Airy (1840), on tracking telescopes and that of Maxwell (1868) on fly-ball governors, control was closely linked to differential equations. These, and other early applications of control technology, were usually concerned with physical systems, modeled in this way. One or two decades later, as the importance of technologies based on the transmission of power and information over electrical networks grew, there emerged a competing way to describe systems based on frequency response and transform methods. The "operational calculus" of Oliver Heaviside eventually led to a distinctively different, "systems" point of view, which often proved to be more practical for these new problems. Eventually these methods were firmly supported by the theory of the Laplace transform and, in time, led to effective ways of thinking about feedback and feedback compensation, in the process developing concepts now often associated with the names Carson, Black, Nyquist and Bode, etc. By the mid 1940s this approach was widely taught in electrical engineering departments.

A decade later the pendulum began to swing the other way. In the 1950s the influential group at RIAS, organized by Lefschetz and LaSalle, played a central role shifting work in America back to the earlier, differential equations dominated, point of view. The RIAS group popularized recent developments in the field of differential equations and control, bringing the considerable body of theory under development in the Soviet Union to the attention of a wider circle of engineers. Especially prominent in this regard were questions related to stability in the sense of Liapunov, including the focus on concrete problems such as the Lur'e problem Aizerman and Gantmcher (1963) and (Lur'e & Postnikov, 1944), relating specifically to nonlinear feedback. This was the Sputnik/Cold War era and developments in the Soviet Union were of great interest, particularly in the United States.[2] The link to technology via control theory helped to revitalize certain problem areas in differential equations and the combination of differential equation methods with frequency response ideas often proved to be remarkably effective in solving concrete problems and explaining their significance in engineering terms. Particularly noteworthy in this regard is the result of Popov–Kalman–Yakubovich on stability (Kalman, 1971; Popov, 1962; Yakubovich, 1962).

2.1. New ideas from differential geometry

Around 1970 the study of nonlinear control systems took a sharp turn. In part, this was driven by the hope for a more inclusive theory which would be applicable to various aerospace problems lying outside the scope of linear theory, and also by the gradual realization that tools from differential geometry, and Lie theory in particular, could be seen as providing a remarkably nice fit with what seemed to be needed for the wholesale extension of linear control theory into a nonlinear setting. For systems describable by differential equations, this geometric approach seemed to hold the promise of a *systematic* development of nonlinear control, something that had been completely missing in the past.[3] Problems such as finding conditions under which the describing function could be validated and understanding the behavior of systems with hysteresis feedback, which had loomed large a decade earlier, suddenly seemed less important in comparison with what could be envisioned with these new methods. However there were impediments. New vocabulary and background material had to be digested, and, in stark contrast to what is available today, the expository literature in the area was sparse and uneven. For this reason the early paper of Hermann (1963), couched as it was in the language of "distributions in the sense of Chevalley" and Jan Kučera's work (Kučera, 1966) using Lie groups, took some time to be appreciated.[4] Indeed, there was a steep learning curve for engineers who wished to follow and contribute to these developments.

The 1972 David Mayne and I set out to improve this state of affairs by organizing a conference at Imperial College that brought together about 100, mostly youngish (≤35) people, with the idea of teaching and learning about how control problems fit in with differential geometric ideas. The lectures ranged in emphasis from applications to abstract theory, touching on a variety of topics. The proceedings (Mayne & Brockett, 1973) represent a faithful account of the lectures but do not fully capture the excitement that went

1 Although rather eclectic, Bellman's book on matrix theory (Bellman, 1955) deserves to be mentioned in this context because of its large number of interesting references and suggestions for further work.

2 Even so, there were important lines of work that did not receive the attention they might have. For example, the body of work on nonholonomic systems discussed in book by Neimark and Fufaev (1972), with its strong geometric flavor and extensive references to the Soviet literature on mechanics, does not appear to have played much of a role.

3 About this time the "geometrization" of classical mechanics, as popularized by the stylish and audacious book of Abraham (with Marsden) (1968), began to attract a larger following and this provided further inspiration. See Brockett (1977).

4 It is somewhat surprising that, notwithstanding the highly influential position Lefschetz held as the algebraic geometer of his day, and the geometric flavor of his book on differential equations (Lefschetz, 1957), I have seen no evidence that he investigated the possibility of using Lie theoretic methods for controllability.