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Adaptive model predictive inventory controller for multiproduct warehouse system[☆]

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ABSTRACT

An inventory control system has been developed for a distribution system consisting of a single multiproduct warehouse serving a set of customers and purchasing products from multiple vendors. Purchase orders requesting multiple products are delivered to the warehouse in a process referred to as joint replenishment. The receipt of customer orders by the warehouse proceeds in order intervals and in order quantities that are subject to random fluctuations. The objective of warehouse operation is to minimize the total cost while maintaining inventory levels within the warehouse capacity by adjusting the purchase order intervals and quantities. An adaptive model predictive control algorithm is developed using a periodic square wave model to represent the material flows. The adaptive concept incorporates a stabilized minimum variance control-type input calculation coupled with input/output stream parameter predictions. The boundedness of the control output under the suggested algorithm is mathematically proven under the assumption that disturbances in the orders are bounded. The effectiveness of the scheme was demonstrated using simulations.

1. Introduction

A prototypical multiproduct distribution system consists of a number of plants or production lines, a warehouse or set of warehouses, and a number of customer supply points. Systems of this type are employed in many sectors of the processing industry, including packaged lubricants, pelletized polymer products, cosmetics and other consumer products. In recent years, the operational emphasis has been on providing fast and timely customer response while maintaining the lowest possible inventory levels in operation warehouses, thus shifting to the production sites the burden of efficient small lot production with rapid product changeovers.

In this work, we address a particular form of this problem in which a single multiproduct warehouse serves a set of customers and purchases products from multiple vendors or production lines. Each of the vendors operates under a periodic schedule with regular lot sizes that can be adjusted as required. Purchasing lots containing multiple products are delivered to the warehouse at fixed intervals using regularly scheduled transport vehicles. Customer orders occur with a fixed order interval and regular order quantity; however, both are subject to considerable random fluctuations. We seek an operational state in which the product inventories within specified upper and lower levels (set band) are maintained, such that a minimum level of safety stock is included, by adjusting the purchasing lot sizes and intervals so as to meet customer requirements. This demand-driven system is posed as a discrete process control problem involving periodic square wave (PSW) input/output flows (Yi & Reklaitis, 1994). Under the proposed feedback control framework, deviations from the expected cycle times and order sizes that generate the demand flows are used to adjust the cycle times and lot sizes of the purchase flows so as to insure that the inventory level of each product neither exceeds the warehouse capacity nor falls below a minimum safety stock level. A model predictive control (MPC) algorithm is developed based on a detailed nonlinear inventory prediction model derived from the basic material balance equations for a system of PSW material flows. The adaptive control algorithm is composed of a stabilized minimum variance type control input calculation coupled with suitable predictions of the input and output streams.

An MPC application for a one-warehouse multi-retailer system has been described previously (Monthatipkula & Yenradee, 2008). This report applied a simple mixed integer linear programming

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(MILP) model with safety stock constraints and compared the performance of the model with the (R, s, S) policy (Hax & Candea, 1984). The applications of MPC techniques to supply chain problems have been briefly reviewed elsewhere (Sarimveisa, Patrinosa, Tarantilisb, & Kiranoudisa, 2008). Perea-López, Ydstie, and Grossmann (2003) developed a model predictive control strategy to find the optimal decision variables to maximize profit in supply chains with multiproduct, multiechelon distribution networks with multiproduct batch plants by using mixed integer linear programming model. They did not consider joint replenishment constraints. Li and Marlin (2009) developed a robust MPC method for a serial supply chain system. Their formulation involved a constrained bi-level stochastic optimization problem that was transformed into a tractable problem involving a limited number of deterministic conic optimization problems. Liu, Shah, and Papageorgiou (2012) proposed an MILP formulation for multi-echelon multiproduct process supply chain planning under demand uncertainty, considering inventory deviations and price fluctuations. Schwartz, Wang, and Rivera (2006) presented a simulation-based optimization framework in which simultaneous perturbation stochastic approximation provided a means for optimally specifying the parameters required for internal model control and MPC-based decision policies for inventory management in supply chains under conditions involving supply and demand uncertainty. Wang and Rivera (2008) developed an MPC-based formulation as a tactical decision tool for supply chain management in semiconductor manufacturing. The features of stochasticity on a short timescale and special requirements arising from the manufacturing process were formally addressed by relying on an observer with multiple-degrees-of-freedom tuning, as well as taking advantage of the flexible constraint handling afforded by MPC.

The complex supply chain network was represented by a batch-storage network (BSN) using a PSW model that covered most supply chain structural components, for example, raw material purchasing, production, transportation, and finished product demand, where the BSN was a general multistage production–inventory system that included the recycling material and financial flows. Optimization models for the BSN and their Kuhn–Tucker solutions have been developed in the context of a variety of network structures, such as a cyclic network, financial transactions, semicontinuous processes, multi-currency flows, and transportation processes (Yi & Reklaitis, 2013). The PSW model used for the supply chain optimization could also be used to design MPC for supply chain facilities, such as a single product storage system (Yi & Reklaitis, 1994). In this study, we will extend the previous study to a multiproduct warehouse system with more functions such as cost reduction and rigorous mathematical development. The MPC design method in this study can be applied directly to the various BSN.

In this study, we assume that vendor's delivery and lead times have no uncertainty, for simple presentation. Note that the control inputs of PSW model, dispatching time and quantity, are directly applied to discrete material flow systems whereas those of existing models are usually flows which need further processing to be applied to the systems.

2. System description

Fig. 1 illustrates the concept of a multiproduct warehouse system. The warehouse delivers multiple products $j = 1, 2, \dots, |J|$ to customers $m = 1, 2, \dots, |M|$ by purchasing the products from vendors $i = 1, 2, \dots, |I|$. Where $|J|$ is the number of elements in product set J, $|M|$ is the number of elements in customer set M, and $|I|$ is the number of elements in vendor set I. The cargo has been ordered and is transported such that multiple products are included within one package, as shown in Fig. 1. Assuming that

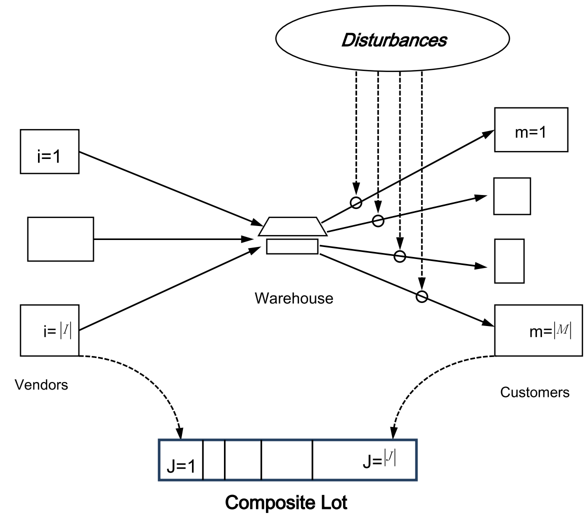

Fig. 1. Multiproduct warehouse system.

the material flows may be described by a PSW, the flows to the warehouse can be represented in terms of two variables: the order size and the cycle time. The disturbances transmitted through the customer demand flows can thus be characterized by the following two equations:

$$\mathbf{B}_m^j = \overline{B_m^j} + \mathbf{e}_{\mathbf{B}_\mathbf{m}^\mathbf{j}} \tag{1}$$

$$\boldsymbol{\omega}_m = \overline{\omega_m} + \mathbf{e}_{\boldsymbol{\omega}_\mathbf{m}} \tag{2}$$

where the bar denotes the expectation operator and the bold character denotes the random variables. The quantities $\mathbf{e}_{\mathbf{B}_\mathbf{m}^\mathbf{j}}$ and $\mathbf{e}_{\boldsymbol{\omega}_\mathbf{m}}$ are noises, $\mathbf{B}_m^j$ is the order size of product j from customer m, $\boldsymbol{\omega}_m$ is the cycle time of the order of customer m. We have one more variable $\mathbf{x}_m$ (≤ 1) which is the storage operation time fraction (SOTF) for the order from customer m. (See Fig. 2 for the definition of SOTF.) The intensity of noises of SOTFs is relatively small and assumed to be zero, for simplicity.

Fig. 2 provides the definitions of the variables and parameters describing the supply (upstream) and demand (downstream) PSW flows. Any of the order sizes (B_i^j), cycle times ($\boldsymbol{\omega}_\mathbf{i}$), and SOTFs ($x_i$) of the supply flows may be used as control variables. From a practical point of view, the SOTFs are not the best control variables and will be assumed to be constant throughout this study. Two types of control inputs are included here: the cycle times (order point), which are the upstream delay time fractions (y_i) determined by our control system variables, and the order sizes (B_i^j). The average flow rate of product j from vendor i (D_i^j) can be used as control input instead of the order size where $B_i^j = D_i^j \omega_i$. The upstream cycle time is assumed to contain slack time in the amount of $z_i\omega_i$, where $z_i\omega_i = \omega_i - \underline{\underline{\omega_i}}$ and $\omega_i \geq \underline{\underline{\omega_i}}$ where $\underline{\underline{\omega_i}}$ is the lower bound of ω_i, which can be used to reduce cycle time length.

The above choice of control variables and parameters was very successful for a single product intermediate storage system (Yi & Reklaitis, 1994). The inventory and flows are assumed to be measurable. The system parameters $\overline{B_m^j}, \overline{\omega_m}, x_m, B_i^j, \omega_i, x_i$ and $\tilde{t}_i$ are treated as known constants, but their values can be updated using updated information, where $\tilde{t}_i$ is the order lead time for vendor i. $\tilde{t}_i$ is a random parameter in the real world, but the randomness is not considered in this study, for simplicity. The main control objective of the warehouse system is that the total inventory should not exceed the upper limit of the warehouse space, and the inventory of each product should not be depleted below the safety stock level