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Inherent robustness properties of quasi-infinite horizon nonlinear model predictive control[☆]

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ABSTRACT

We consider inherent robustness properties of model predictive control (MPC) for continuous-time nonlinear systems with input constraints and terminal constraints. We show that MPC with a nominal prediction model and persistent but bounded disturbances has some degree of inherent robustness when the terminal control law and the terminal penalty matrix are chosen as the linear quadratic control law and the related Lyapunov matrix, respectively. We emphasize that the input constraint sets can be any compact set rather than convex sets, and our results do not depend on the continuity of the optimal cost function or of the control law in the interior of the feasible region.

1. Introduction

Model predictive control (MPC) has received remarkable attention in both practical applications and theoretical research over the last 30 years since it is capable of explicitly dealing with state and input constraints (Mayne, Rawlings, Rao, & Scokaert, 2000; Qin & Badgwell, 2003). The basic idea of standard MPC (Chen & Allgöwer, 1998; Fontes, 2001; Magni, De Nicolao, & Scattolini, 2001; Mayne et al., 2000) is as follows: Online, a finite horizon open-loop optimal control problem based on the current measurement of the system states is solved. Then, the first part of the obtained open-loop optimal input trajectory is applied to the system. At the succeeding time instant, the optimal control problem is solved again using new state measurements and with a shifted horizon, and the actual control input is updated.

For a nominally stabilizing model predictive control (MPC) scheme the presence of disturbances and/or model uncertainties may lead to performance deterioration or even loss of stability. An intuitive approach to guarantee robust stability and recursive feasibility is to use a min–max MPC formulation, where the optimal input is determined such that the performance criteria is minimized for the worst-case uncertainty (Bemporad, Borrelli, & Morari, 2003; Chen, Scherer, & Allgöwer, 1997; Fontes & Magni, 2003; Limon, Alamo, Salas, & Camacho, 2006; Magni, De Nicolao, Scattolini, & Allgöwer, 2003; Raimondo, Limon, Lazar, Magni, & Camacho, 2009; Scokaert & Mayne, 1998). However, such approaches are usually computationally expensive. Furthermore, the optimal input is obtained for a possibly unrealistic worst-case scenario, which often results in poor performance in the case of small actual uncertainties. Constraint tightening approaches, as introduced by Chisci, Rossiter, and Zappa (2001), Limon, Alamo, and Camacho (2002) and Richards and How (2006), can avoid computational complexity by using a nominal prediction model and tightened constraint sets. However, the constraint sets shrink drastically because the "margin", which reflects the effect of uncertainties, increases exponentially with the increase of the prediction horizon. For linear discrete-time systems with persistent disturbances, Mayne, Seron, and Rakovic (2005) and Rawlings and Mayne (2009) provide a new constraint tightening, tube based robust MPC scheme, which has fixed tightened sets. The results utilize both state feedback control and feedforward control action, and have been extended by Rakovic, Teel, Mayne, and Astolfi (2006); Yu,

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Böhm, Chen, and Allgöwer (2010) to systems with matched nonlinearity and piecewise affine systems. The tube based robust MPC scheme has also been extended to general discrete-time nonlinear systems (Mayne, Kerrigan, van Wyk, & Falugi, 2011). It possesses two loops, where a nominal MPC scheme in the inner loop generates a reference trajectory and the MPC control in the outer loop steers trajectories of the uncertain systems towards the reference trajectory. The schemes are based on the *a priori* estimation of the effect of disturbances over the prediction horizon.

Since robust MPC methods are much more complex than those developed for the nominal case, it is of interest to analyze under which conditions nominal MPC can guarantee robustness with respect to specific classes of disturbances. The paper (Grimm, Messina, Tuna, & Teel, 2004) used examples to illustrate that MPC applied to nonlinear systems can produce nominal asymptotic stability without any robustness, when the optimization problem contains state constraints or equality terminal constraints. In the examples, either the considered nonlinear system is discontinuous at its equilibrium or the Jacobian linearization of the considered nonlinear systems is not stabilizable. Under the fundamental assumption that the presence of uncertainties or disturbances do not cause any loss of feasibility, robustness properties of nominal MPC algorithms are proved in Magni and Sepulchre (1997), Nicolao, Magni, and Scattolini (1996) and Scokaert, Rawlings, and Meadows (1997). The recursive feasibility assumption holds true when the problem formulation does not include state and input constraints and when the terminal constraint used to guarantee nominal stability is also satisfied under perturbed conditions (Magni & Scattolini, 2007). For unconstrained input-affine nonlinear systems, it is shown in Magni and Sepulchre (1997) that the nominal MPC control law is inverse optimal. Thus, it is also optimal for a modified optimal control problem spanning over an infinite horizon. Due to this inverse optimality property, the MPC control law inherits the same robustness properties as the infinite horizon optimal control assuming that the sampling time goes to zero. Under the assumption that the optimal cost function is twice continuously differentiable, it has been shown in Nicolao et al. (1996) that MPC control law provides robustness with respect to gain perturbations due to actuator and additive perturbations describing unmodeled dynamics. Results on inherent robustness with exponentially decaying disturbances are reported in Scokaert et al. (1997) with the assumption that the MPC control law is Lipschitz continuous. The papers (Findeisen & Allgöwer, 2005; Limon et al., 2009; Pannocchia, Rawlings, & Wright, 2011) show that nominal MPC possesses inherent robustness properties if the optimal cost function is locally Lipschitz continuous or the MPC control law is regionally continuous. However, both the resulting MPC control law and the optimal value function associated to the optimization problem defining nominal MPC can be discontinuous (Fontes, 2000; Meadows, Henson, Eaton, & Rawlings, 1995; Rawlings & Mayne, 2009). While MPC is applied to linear systems with convex constraints, some robustness exists (Grimm et al., 2004). The result depends on the fact that continuity of the optimal value function on the interior of the feasible region is a sufficient condition for robustness, as is continuity of the feedback law on the interior of the feasible region (Jiang & Wang, 2001). The paper (Grimm, Messina, Tuna, & Teel, 2007) shows that the system under control is robust to sufficient small disturbances, if (a) the value function is bounded by a $\mathcal{K}_\infty$ function of a state measure (related to the distance from the state to some target set) and this measure is detectable from the stage cost used in the MPC algorithm; (b) the systems satisfy a definition that attempts to characterize the robustness properties of the MPC optimization problem. Instead of the analysis of the inherent robustness properties of existing nominal MPC schemes, Lazar and Heemels (2009) and Picasso, Desiderio, and Scattolini (2010, 2011, 2012) propose novel nominal MPC schemes which have some inherent robustness properties.

Quasi-infinite horizon MPC (Chen & Allgöwer, 1998; Mayne et al., 2000) is one of the main results of nonlinear MPC with guaranteed nominal stability. Our previous conference paper (Yu, Reble, Chen, & Allgöwer, 2011) considers the inherent robustness properties of quasi-infinite horizon MPC with input constraints and a terminal constraint. Although the recursive feasibility is proved directly, the proof of robust stability is not complete. In this paper, we rigorously show inherent robustness properties of quasi-infinite horizon MPC of nonlinear systems with input constraints, where the disturbances are persistent but bounded and the optimization problem has a terminal constraint. It is worth noting that the following analysis does neither assume the continuity of the optimal cost function nor of the control law, and hence the results are more general than previous results available in the literature. It is shown that the degree of robustness depends on the terminal set and on the terminal penalty function, the prediction horizon, the upper bound on the disturbances, and the Lipschitz constant of the system.

The remainder of the paper is organized as follows. The problem is set up in Section 2. Terminal conditions for nominal stability, recursive feasibility of the online optimization problem, and robust stability are proposed in Section 3. Further results on inherent robustness properties of linear MPC is discussed in 4. Section 5 provides two examples to demonstrate the effectiveness of the derived results.

1.1. Notations and basic definitions

Let $\mathbb{R}$ denote the field of real numbers and $\mathbb{R}^n$ the n-dimensional Euclidean space, $\mathbb{Z}_{[0,\infty)}$ the field of non-negative integers. For a vector $v \in \mathbb{R}^n$, $\|v\|$ denotes the 2-norm and $\|v\|_Q = \sqrt{v^TQv}$ with $Q \in \mathbb{R}^{n\times n}$ and $Q > 0$. Let $M \in \mathbb{R}^{n\times n}$, $\lambda_{\min}(M)$ ($\lambda_{\max}(M)$) is the smallest (largest) real part of the eigenvalues of matrix M and $\sigma(M)$ the largest singular value of M. The operation $\oplus$ is the addition of sets $\mathcal{A} \subset \mathbb{R}^n$ and $\mathcal{B} \subset \mathbb{R}^n$, $\mathcal{A} \oplus \mathcal{B} := \{a + b \in \mathbb{R}^n | a \in \mathcal{A}, b \in \mathcal{B}\}$. The operation $\ominus$ is the subtraction of sets $\mathcal{A} \subset \mathbb{R}^n$ and $\mathcal{B} \subset \mathbb{R}^n$, where $\mathcal{A} \ominus \mathcal{B} := \{x \in \mathbb{R}^{n_x} | \{x\} \oplus \mathcal{B} \subseteq \mathcal{A}\}$. Denote the set $\mathcal{B}(x_0, \delta) := \{x \in \mathbb{R}^n \mid \|x - x_0\| \le \delta\}$, $\mathcal{B}(\delta) := \{x \in \mathbb{R}^n \mid \|x\| \le \delta\}$, and $\varnothing$ as the empty set. Denote $\mathbb{L}^n_{[a,b]}$ as the space of all Lebesgue functions mapping from $[a, b]$ to $\mathbb{R}^n$.

We introduce the following definitions which will be used in the paper:

Definition 1. A system is ultimately bounded if the system converges asymptotically to a bounded set.

Definition 2 (*Hausdorff Distance Rawlings & Mayne, 2009*)**.** The Hausdorff distance $d(\cdot, \cdot)$ between two sets $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Y} \subset \mathbb{R}^n$ is defined by

$$d(\mathcal{X}, \mathcal{Y}) := \max\left\{\sup_{x\in\mathcal{X}} d(x, \mathcal{Y}),\ \sup_{y\in\mathcal{Y}} d(y, \mathcal{X})\right\},$$

in which $d(a, \mathcal{M})$ denotes the distance of a point $a \in \mathbb{R}^n$ from a set $\mathcal{M} \subset \mathbb{R}^n$, which is defined by

$$d(a, \mathcal{M}) := \inf_{b\in\mathcal{M}} d(a, b),$$

where $d(a, b) = \|a - b\|$.

Definition 3 (*Relation*)**.** Suppose that both $\mathcal{X}$ and $\mathcal{Y}$ are compact sets with $\mathcal{X} \subseteq \mathcal{Y} \subset \mathbb{R}^n$, and $\bar{\mathcal{X}}$ and $\bar{\mathcal{Y}}$ are the boundaries of sets $\mathcal{X}$ and $\mathcal{Y}$, respectively. The relation $d_r(\cdot, \cdot)$ between sets $\mathcal{X}$ and $\mathcal{Y}$ is defined by

$$d_r(\mathcal{X}, \mathcal{Y}) := \min_{x\in\bar{\mathcal{X}},\, y\in\bar{\mathcal{Y}}} \|x - y\|.$$