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Optimally conditioned instrumental variable approach for frequency-domain system identification☆

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ABSTRACT

Accurate frequency-domain system identification demands for reliable computational algorithms. The aim of this paper is to develop a new algorithm for parametric system identification with favorable convergence properties and optimal numerical conditioning. Recent results in frequency-domain instrumental variable identification are exploited, which lead to enhanced convergence properties compared to classical identification algorithms. In addition, bi-orthonormal polynomials with respect to a data-dependent bi-linear form are introduced for system identification. Hereby, optimal numerical conditioning of the relevant system of equations is achieved. This is shown to be particularly important for the class of instrumental variable algorithms, for which numerical conditioning is typically quadratic compared to alternative frequency-domain identification algorithms. Superiority of the proposed algorithm is demonstrated by means of both simulation and experimental results.

1. Introduction

Frequency-domain system identification (McKelvey, 2002; Pintelon & Schoukens, 2001) is of significant importance for a broad class of applications, since it enables (i) straightforward data reduction, (ii) straightforward combination of multiple data sets, (iii) direct estimation and use of nonparametric noise models, and (iv) a direct connection to control-relevant identification criteria.

Many parametric identification techniques based on frequency-domain data involve a nonlinear least-squares problem. Here, the nonlinearity arises from the parametrization of the poles in the denominator polynomial. In Levy (1959), the nonlinear problem is approximated using a single linear least-squares problem. However, this introduces an *a priori* unknown weighting function. The SK-algorithm (Sanathanan & Koerner, 1963) mitigates the effect of such weighting through iterations. In Bayard (1994) and de Callafon, de Roover, and Van den Hof (1996), the SK-algorithm is generalized to multivariable systems. Nevertheless, two aspects require further attention.

On one hand, frequency-domain identification problems are typically numerically ill-conditioned. Several partial solutions exist, including (i) frequency scaling (Pintelon & Kollár, 2005), (ii) amplitude scaling (Hakvoort & Van den Hof, 1994), and (iii) the use of orthonormal polynomials and orthonormal rational functions with respect to a continuous inner product, see, e.g., Heuberger, Van den Hof, and Wahlberg (2005) and Ninness and Hjalmarsson (2001) for a connection with numerical properties. These approaches confirm that ill-conditioning is an important aspect in system identification applications and they typically improve numerical conditioning. However, these partial solutions may be insufficient to reliably solve complex frequency-domain identification problems. Therefore, in Oomen and Steinbuch (to appear) and van Herpen, Oomen, and Bosgra (2012b), an approach is presented that leads to optimal numerical conditioning of the SK-algorithm by using polynomials that are orthonormal with respect to a data-based discrete inner product, see Reichel, Ammar, and Gragg (1991) and Van Barel and Bultheel (1995) for a definition and earlier results.

On the other hand, the fixed point of the SK-algorithm generally does not correspond to a (local) minimum of the nonlinear least-squares criterion, as shown in Whitfield (1987). Consequently, the

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SK-algorithm is typically used as an initialization for subsequent Gauss–Newton iterations, see, e.g., Bayard (1994) and Pintelon and Schoukens (2001, Section 7.9.1), which guarantees convergence to a (local) minimum.

Recently, in Douma (2006, Sections 3.5.3 and 3.5.8), an alternative frequency-domain identification algorithm has been formulated, in which a fixed point of the iterations corresponds to an optimum of the objective function. This renders a Gauss–Newton iteration superfluous, potentially enabling an increase of algorithm efficiency. The new algorithm, which has been extended towards multivariable systems in Blom and Van den Hof (2010), takes the form of an iterative instrumental variable method, see also Stoica and Söderström (1981) and Young (1976) for earlier results in this direction.

Although the result in Blom and Van den Hof (2010) and Douma (2006) potentially reduces the number of iterations in frequency-domain identification, a direct implementation of the algorithm exhibits poor numerical properties. This is further supported in this paper, both by means of a theoretical analysis and a numerical example. In fact, the condition numbers associated with the algorithm are quadratically larger than for the standard SK-iterations. This obstructs a reliable and accurate computation of the optimal model. In addition, the approach in Reichel et al. (1991), Van Barel and Bultheel (1995) and van Herpen et al. (2012b) for optimal conditioning of the SK-iterations does not apply to the algorithm in Blom and Van den Hof (2010) due to the lack of an appropriate inner product.

The main contribution of this paper is a new framework for frequency-domain system identification based on a nonlinear least-squares criterion, which (i) provides advantageous convergence properties, and (ii) ensures optimal numerical conditioning ($\kappa = 1$). Essentially, the proposed solution exploits the results in Blom and Van den Hof (2010), while providing optimal numerical conditioning in the spirit of Reichel et al. (1991), Van Barel and Bultheel (1995) and van Herpen et al. (2012b), albeit through a fundamentally different mechanism. In particular, the new algorithm relies on the introduction of *bi-orthonormal polynomial bases* in system identification. Recently, in Gilson, Welsh, and Garnier (2013) and Welsh and Goodwin (2003), the need for enhancement of numerical conditioning in frequency-domain instrumental variable identification has been confirmed and some enhancements have been obtained by using an alternative polynomial basis. The approach in this paper reformulates the instrumental-variable algorithm using bi-orthonormal polynomials with respect to a *data-dependent bi-linear form*, which leads to optimal numerical conditioning, i.e., $\kappa = 1$. The following specific contributions are presented in this paper.

(C1) The numerical conditioning that is associated with the linear system of equations of the algorithm in Blom and Van den Hof (2010) is quadratically larger than the condition numbers encountered in the standard SK-iterations (Bayard, 1994; de Callafon et al., 1996; Sanathanan & Koerner, 1963), as is shown both theoretically and by means of a numerical example.

(C2) The algorithm in Blom and Van den Hof (2010) has the interpretation of an instrumental variable method. Such type of method admits a transformation of instruments (Söderström & Stoica, 1983). This freedom is exploited to formulate the algorithm in two distinct polynomial bases: one for the model (at the present iteration) and one for the instrument.

(C3) Optimal numerical conditioning is achieved by selecting polynomial bases that are bi-orthonormal with respect to a data-dependent bi-linear form. This bi-linear form accounts for the asymmetric and indefinite character of instrumental variable problems. As a special case, the bi-orthonormal polynomial bases include the orthonormal polynomials with respect to a data-based discrete inner product in Reichel et al. (1991), Van Barel and Bultheel (1995) and van Herpen et al. (2012b).

(C4) Identification of a SISO rational transfer function requires modeling of a numerator and denominator polynomial. Thus, this paper considers a 2×1 vector-polynomial, which is developed in terms of a 2×2 block-polynomial basis. The construction of bi-orthonormal block-polynomials from given frequency response data is presented for continuous-time systems. It is shown that an efficient construction using three-term-recurrence relations is possible, where the recursion coefficients are obtained from a matrix 2×2 block-tridiagonalization problem.

(C5) Superiority of the proposed algorithm is shown by means of a simulation example and is experimentally validated on an industrial motion system.

This paper extends the results in van Herpen, Oomen, and Bosgra (2012a), in which optimal conditioning of asymmetric polynomial equalities using bi-orthonormal polynomials is introduced, by (i) explicitly connecting bi-orthonormal polynomials with instrumental variable identification (C2)–(C3), and (ii) extending the construction of scalar bi-orthonormal polynomials towards 2×2 block-polynomials (C4). The latter result facilitates the estimation of a numerator–denominator *vector-polynomial*, which enables a confrontation of the proposed method with frequency-domain identification problems (C5). In van Herpen (2014, Chap. 2), a complementary study of relevant aspects in the theory of bi-orthonormal polynomials is provided.

This paper is organized as follows. In Section 2, the frequency-domain identification problem is posed and two iterative algorithms are compared with respect to their convergence properties. In Section 3, the numerical properties of both algorithms are evaluated, motivating the need for enhancement of numerical conditioning (C1). Then, in Section 4, bi-orthonormal polynomials are introduced in frequency-domain system identification, which provides optimal numerical conditioning (C2)–(C3). Subsequently, in Section 5, the construction of bi-orthonormal polynomials using three-term-recurrence relations is presented (C4). In Section 6, an experimental validation of the benefits of the new algorithm for frequency-domain system identification is provided (C5). Conclusions are drawn in Section 7.

Notation: throughout this paper, ξ represents either $s = \jmath\omega$ or $z = e^{\jmath\omega}$, $\jmath = \sqrt{-1}$, where $\omega \in \mathbb{R}$ denotes a frequency. Moreover, $\mathbb{R}^{p\times q}[\xi]$ denotes a $p \times q$ matrix of real polynomials in ξ. Finally, A^* denotes the conjugate of A, whereas A^H denotes the conjugate transpose of A.

Scope: for clarity of the exposition, attention is restricted to identification of SISO systems. The results in this paper can be generalized to the multivariable situation along conceptually similar lines. In this case, matrix fraction descriptions (MFDs), see, e.g., Kailath (1980, Chap. 6), provide a suitable framework as they directly connect to state-space models. Note that besides model order selection, i.e., the McMillan degree of the model, such multivariable models also require the selection of Kronecker indices. The reader is referred to Gevers (1986) and Moore (1981) for further information.

In the second part of the paper, the construction of bi-orthonormal block-polynomial bases is presented. Here, attention is restricted to polynomials in the s-domain. The construction of bi-orthonormal block-polynomial bases in the z-domain is conceptually similar.