Automatica 50 (2014) 2321–2326

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

An invariant observer for a chemostat model[☆]

Ibtissem Didi[a,1], Hacen Dib[b], Brahim Cherki[a]

[a] Department of Electrical Engineering and Electronics, Tlemcen University, Algeria

[b] Department of Mathematics, Tlemcen University, Algeria

ARTICLE INFO

Article history:
Received 5 March 2013
Received in revised form
16 April 2014
Accepted 5 May 2014
Available online 10 August 2014

Keywords:
Chemostat model
Nonlinear systems
Invariant systems
Invariant observers
Lie groups

ABSTRACT

In this paper, we tackle the problem of designing an invariant observer for a chemostat model with adjustable and robust convergence. The main idea of the paper is to build a new class of observers for chemostat model using hidden symmetries. The effectiveness of the proposed observer is shown through simulations.

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1. Introduction

In bioprocesses, direct measurement of some variables requires expensive sensors which may not exist. To overcome these problems, one uses software sensors also called observers. In most cases the technology used in industry to solve these challenges is based on the *Extended Kalman Filter* (Bastin & Dochain, 1990).

Many types of observers for bioreactors have been proposed and studied by Bastin and Dochain (1990), using the Extended Kalman Filter and the *Extended Luenberger observer*. These approaches are well understood and only require linearization of the model which results in local convergence. They also proposed *asymptotic observers* (Bastin & Dochain, 1990) for relatively simple systems, extended by Chen (1992), to more complex ones. The design of such observers is quite simple, and requires only partial knowledge of the model. Unfortunately, the speed of convergence is not adjustable, and the change of coordinates used in these methods depends on the stoichiometric coefficients which makes these observers not robust to the variation of parameters. Some authors proposed *high-gain observers* (Gauthier, Hammouri, & Othman, 1992) based on a nonlinear change of coordinates to put the system in a canonical observer form. For such observers a major drawback is the sensitivity to the noise measurement. Other authors proposed a new kind of observers called *interval observers*. These are designed for systems with large uncertainties (Gouze, Rapaport, & Hadj-Sadok, 2000). They consist of the association of two observers, one to observe the lower bound and another to observe the upper bound of the states. Moreover, the system must satisfy a rather strong property called cooperativity, and it is necessary to know the bounds of the uncertainties in the model.

A new approach to synthesize nonlinear observer based on Lie groups of symmetries was developed by Rouchon, Bonnabel, and Martin (2008). This kind of observers, called *invariant observers*, can be constructed in a canonical way and have a general gain form. These observers were applied to Lagrangian systems, chemical exothermic reactors, inertial navigation and polymerization reactors (Aghannan, 2003; Aghannan & Rouchon, 2003; Bonnabel, 2007; Bonnabel, Martin, & Rouchon, 2006, 2008).

This paper is organized as follows: In Section 2 we recall all the needed definitions about invariant systems and invariant observers. Section 3 is devoted to the design of an invariant observer for the chemostat model. In Section 4 we state the main result of this paper. Finally, in Section 5 we give some simulations to show the effectiveness of the proposed method.

2. Invariant systems

The concept of invariance used in this paper is the invariance under a group action. This concept has been already used by

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[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Jun-ichi Imura under the direction of Editor Toshiharu Sugie.

E-mail addresses: didi.ibtissem@gmail.com (I. Didi), h_dib@mail.univ-tlemcen.dz (H. Dib), cherki@mail.univ-tlemcen.dz (B. Cherki).

[1] Tel.: +213 553571978; fax: +213 43213198.

http://dx.doi.org/10.1016/j.automatica.2014.07.005
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many authors especially by Bonnabel (2007), Rouchon (2009) and Rouchon et al. (2008) in the context of Control Theory. For a more general setting, one can see Olver's books (Olver, 1993, 1995).

Consider the dynamical system (1)–(2), where u is the input vector of the system, x the state vector and y the measured output:

$$\dot{x} = f(x, u) \tag{1}$$

$$y = h(x, u) \tag{2}$$

with $x \in X \subset R^n$, $u \in U \subset R^m$ and $y \in Y \subset R^p$.

Let G be a Lie group of transformations, G acts on X by $\varphi_g : X \to X$ $\forall g \in G$, φ_g is a diffeomorphism (at least C^1) on X with $(\varphi_g)^{-1} = \varphi_{g^{-1}}$ and $\varphi_{g_1} \circ \varphi_{g_2} = \varphi_{g_1 . g_2}$. We consider also the action of the same group G on U by $(\psi_g)_{g\in G}$ and on Y by $(\rho_g)_{g\in G}$.

Definition 1. *G is a symmetry group of (1) if, for every solution $(x(t), u(t))$ of (1) and $\forall g \in G$, $\left(\varphi_g(x(t)), \psi_g(u(t))\right)$ is also a solution.*

Consequently, the system (1) is said to be *invariant* under G iff:

$$f\left(\varphi_g(x), \psi_g(u)\right) = D_{\varphi_g}(x) f(x, u), \quad \text{for all } g, x \text{ and } u$$

where D_{φ_g} is the Jacobean matrix of $\varphi_g(x)$.

The output y is said to be *equi-variant* iff:

$$h\left(\varphi_g(x), \psi_g(u)\right) = \rho_g(h(x, u)), \quad \text{for all } g,\ x \text{ and } u.$$

The dynamical system $\frac{d}{dt}\hat{x} = F(\hat{x}, u, \hat{y})$ is a *pre-observer* of the system (1)–(2) iff:

$$F(x, u, h(x, u)) = f(x, u), \quad \text{for all } x \text{ and } u$$

and is said to be *invariant* iff:

$$F\left(\varphi_g(\hat{x}), \psi_g(u), \rho_g(\hat{y})\right) = D_{\varphi_g}(\hat{x}) F(\hat{x}, u, \hat{y})$$

for all g, $\hat{x}$, u and $\hat{y}$.

2.1. General form of the invariant pre-observers

To build such an observer, we need two important ingredients: *scalar invariant functions* and *invariant vectors fields*.

A function J defined on a domain X, is said to be *invariant* iff:

$$J(\varphi_g(x)) = J(x) \quad \text{for all } g \text{ and } x.$$

A vector field w is said to be *invariant* with respect to the group action φ_g on $X \subset R^n$ iff:

$$w(\varphi_g(x)) = D_{\varphi_g}(x) w(x) \quad \text{for all } g \text{ and } x.$$

It is not hard to prove that the dynamical system:

$$\frac{d}{dt}\hat{x} = f(\hat{x}) + \sum_i J_i(\hat{x}, y) w_i(\hat{x}) \tag{3}$$

is an *invariant pre-observer* for the system (1)–(2) (Rouchon et al., 2008), if we choose J_i as a scalar invariant function satisfying $J_i\left(\hat{x}, f(\hat{x}, u)\right) = 0$ and w_i as invariant vectors fields. This formula expresses the general form of an invariant pre-observer. If the invariant pre-observer (3) converges to the model (1), then (3) is called an invariant observer. By convergence we mean

$$\lim_{t\to+\infty} \operatorname{dist}(\hat{x}(t), x(t)) = 0$$

where "dist" is a distance measure on the state space.

3. An Invariant observer for the chemostat model

The chemostat is a kind of bioreactor, which was introduced by Novick and Szilard (1950) and used by Monod (1950).

Generally, this device is used for the growth of bacteria, phytoplankton, etc. It works in continuum mode, i.e, the volume of the bioreactor is kept constant. It is a laboratory prototype of bioreactors used in waste water treatment.

The nonlinear model of the chemostat obtained by mass balance is given by:

$$\begin{aligned} \dot{s}(t) &= D_l(t)\,(s_{in} - s(t)) - k\mu\,(s(t), K)\,x(t) \\ \dot{x}(t) &= [\mu\,(s(t), K) - D_l(t)]\,x(t) \end{aligned} \tag{4}$$

where $s \in R_+$ and $x \in R_+$ represent the substrate concentration and the biomass concentration respectively, $D_l(t) > 0$ is the dilution rate, $0 < D_{l,min} < D_l(t) < D_{l,max}$, k is the growth yield, s_{in} is the input substrate concentration and $\mu\,(s, K)$ is the specific growth rate per unit of biomass. In our paper the Monod specific function is used for $\mu\,(s, K)$:

$$\mu\,(s(t), K) = \mu_{max}\frac{s(t)}{s(t) + K}$$

where μ_{max} is the maximum growth rate and K is the half saturation constant.

In the sequel we will consider s as the measured variable: $y = s$.

The model is algebraically observable since from $\dot{y}$ (Conte, Moog, & Perdon, 2007), we can deduce the unmeasured state variable x.

Concerning the positivity of the state variables, it was proved in Smith and Waltman (1995) that if the initial conditions of the system (4) are $s(0) > 0$ and $x(0) > 0$, then the trajectories lie in the orthant $R_+^* \times R_+^*$ for all future times.

The most important contribution of this work is to bring out symmetries which are not obvious since the system was obtained by mass balance law. Our aim is to use these symmetries to build an observer keeping the nonlinear structure of the original system and improve the convergence rate. Consider $G = (R_+^* \times R_+^*, .)$, G acts as "homothety" group with two parameters (λ_1, λ_2) acting on (s, x) which represent the state variable.

Generally, it is not easy to show that a differential system has symmetries. In our case, and relying on Aghannan's ideas (Aghannan, 2003), we choose some variables and let them play the role of a virtual input in order to force the system to be invariant under the "homothety's" action.

Let $\upsilon = (s_{in}, k, K)$ be the virtual control used to display symmetries in the system. The only input of the system is s_{in}. Since k and K are constant parameters, we will keep them at constant values as part of the virtual control.

The actions of G on X, U and Y are defined by:

$$\varphi_{\lambda_1,\lambda_2}(s, x) = (\lambda_1 s, \lambda_2 x)$$

$$\psi_{\lambda_1,\lambda_2}(s_{in}, k, K) = \left(\lambda_1 s_{in}, \frac{\lambda_1}{\lambda_2}k, \lambda_1 K\right)$$

$$\rho_{\lambda_1,\lambda_2}(y) = \lambda_1 y.$$

Let $f_1(s, x, s_{in}, k, K)$ and $f_2(s, x, s_{in}, k, K)$ be the right hand parts of (4). Let us verify that this system is invariant under G-action. We have on one hand:

Let us consider f_1 and f_2 as two functions with five arguments, as follows:

$$\begin{aligned} f_1\left(\lambda_1 s, \lambda_2 x, \lambda_1 s_{in}, \frac{\lambda_1}{\lambda_2}k, \lambda_1 K\right) &= D_l(t)(\lambda_1 s_{in} - \lambda_1 s) \\ &\quad - \frac{\lambda_1}{\lambda_2}k\mu(\lambda_1 s, \lambda_1 K)\lambda_2 x \\ &= \lambda_1 f_1(s, x, s_{in}, k, K) \end{aligned}$$

$$\begin{aligned} f_2\left(\lambda_1 s, \lambda_2 x, \lambda_1 s_{in}, \frac{\lambda_1}{\lambda_2}k, \lambda_1 K\right) &= [\mu(\lambda_1 s, \lambda_1 K) - D_l(t)]\lambda_2 x \\ &= \lambda_2 f_2(s, x, s_{in}, k, K) \end{aligned}$$

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