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Brief paper

Exponential consensus of general linear multi-agent systems under directed dynamic topology☆

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ABSTRACT

This paper aims to investigate the consensus control of generic linear multi-agent systems (MASs) under directed dynamic topology. Nonnegative matrix theory, in particular the product properties of infinite row-stochastic matrices, which are widely used for multiple integrator agents, is explored to deal with the convergence analysis of generic linear MASs. It is finally shown that the exponential consensus can be reached under very relaxed conditions, i.e., the directed interaction topology is only required to be repeatedly jointly rooted and the exponentially unstable mode of each individual system is weak enough. Moreover, a least convergence rate and a bound for the unstable mode of the individual agent system, both of which are independent of the switching mode, can be explicitly specified.

1. Introduction

Last decade has witnessed the rapid development of consensus problem for multiple integrator agents from various different perspectives, see, e.g. Cai and Ishii (2011), Cao, Morse, and Anderson (2008a), Gao and Wang (2011), Jadbabaie, Lin, and Morse (2003), Li and Zhang (2010), Lin, Francis, and Maggiore (2005, 2007), Qin and Gao (2012), Ren and Beard (2005), Wen, Duan, Yu, and Chen (2012), Xiao and Wang (2008), Xia and Cao (2011) and Yu, Chen, and Cao (2010). Consensus problem refers to the problem of multiple agents with the collective objective of reaching agreement about some variables of interests such as attitude, position, velocity, and voltage. It plays an important role in achieving distributed multi-agent coordination.

One of the key problems in multi-agent consensus is finding the weakest possible condition that need to be imposed on the interaction topology to guarantee the desired consensus behavior. It is well known that to reach the consensus, having a rooted graph (also called having a directed spanning tree) is the weakest condition Ren and Beard (2005) on the fixed directed interaction topology, while having a repeatedly jointly rooted interaction topology is the weakest possible condition for the dynamic case Jadbabaie et al. (2003), Moreau (2005) and Ren and Beard (2005). Nonnegative matrix theory, in particular the product properties of infinite row-stochastic matrices Wolfowitz (1963), is demonstrated being the most effective and popular analysis tool in dealing with the convergence analysis for a group of linearly coupled agents under dynamic topology Jadbabaie et al. (2003), Qin and Gao (2012), Ren and Beard (2005, 2008) and Xiao and Wang (2008). One main reason is that in most of the existing literature concerning consensus of integrator agents, agents usually have no dynamics in the absence of interactions and it is the agent interaction only that determines the evolution of agents. As such, system matrix takes a relatively simple form which can be transformed to be a row-stochastic matrix for consideration.

Very recently, much attention is switched to the coordination control of generic linear MASs which takes more general form and thus can be used to model more complicated scenarios in real applications. In this framework, each agent, referred to as generic linear agent for clarity, is modeled by the following dynamics:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the ith agent's state, $B \in \mathbb{R}^{n\times m}$, $u_i \in \mathbb{R}^m$ is the control input for agents i which uses only the local state information from its neighboring agents. Different from the integrator agents,

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the collective behavior of generic linear MASs is determined not only by the dynamical rules governing the isolated agents, modeled by e.g., $\dot{s}(t) = As(t)$ in this paper, but also by the interactions between the neighboring agents. This makes most of the analysis tools including nonnegative matrix theory cannot be extended straightforwardly to generic linear MASs. As compared to the integrator agents, coordination control for generic linear agents is much more challenging. Some efforts made towards tackling the consensus problem of system (1) by using static distributed feedback controller can be found in, e.g., Ma and Zhang (2010), Seo, Shim, and Back (2009) and Tuna (2009) under fixed topology and Ni and Cheng (2010), Tuna (2008) and Wang, Chen, and Hu (2008) under switching topology.

Consensus/synchronization under fixed interaction topology is investigated in Tuna (2008) and You and Xie (2011) in discrete-time setting and in Qin, Gao, and Zheng (2014) and Tuna (2009) in continuous-time setting. Similar to that required for integrator agents, having a rooted graph is shown to be the weakest condition that should be imposed on the interaction topology to guarantee the consensus. Considering, as mentioned earlier, the analysis techniques for integrator agents cannot be extended straightforwardly to the generic linear agents, Ni and Cheng (2010) and Wang et al. (2008) investigate the consensus problem for generic linear MASs under undirected dynamic topology using developed Lyapunov stability theory which, however, relies heavily on the symmetric property of the associated Laplacian matrix, and thus making it invalid in dealing with directed interaction topology setting. Specifically, interaction topology is required to be frequently connected in Wang et al. (2008), and this restrictive condition is later relaxed to a jointly connected one in Ni and Cheng (2010), which is only valid in a leader-following framework. One can easily observe from Ni and Cheng (2010) that the proof technique does not work for the case without a leader.

Noting on one hand that real-world information exchanges between agents may be unidirectional and thus results in the directed interaction topology, and on the other hand that the leader-following scenario is a special case of the leaderless consensus to be considered, this paper aims to investigate the consensus control of generic linear MASs under a very general setting, i.e. leaderless consensus control under directed dynamic topology. The contribution of our work comparing to the existing ones comprise mainly the following two points:

(a) Totally different convergence analysis for generic linear MASs from the existing ones is used in our work. We attempt to bridge the gaps between the consensus analysis for integrator agents, see e.g. Jadbabaie et al. (2003), Qin and Gao (2012), Ren and Beard (2005), Xiao and Wang (2008), and that for generic linear agents under dynamic topology. Toward this end, nonnegative matrix theory, in particular the product properties of infinite row-stochastic matrices, is developed combined with some other matrix analysis techniques to deal with consensus of generic linear MASs. It is worth mentioning that although nonnegative theory has been widely used in the convergence analysis of multiple integrator agents, this work, to the best of our knowledge, is one of the first attempts to employ nonnegative matrix theory to deal with general linear MASs. Moreover, instead of performing only the asymptotical consensus analysis, we consider in this paper the exponential consensus and further specify a least convergence rate under dynamic topology. This generalizes the existing convergence analysis in Jadbabaie et al. (2003), Qin and Gao (2012), Ren and Beard (2005) and Xiao and Wang (2008) to very general settings. These can also be included as the main reasons for not taking a Lyapunov stability related analysis. In general, the consensus value relates to not only the initial states of the agents but also how the interaction topology switches. Besides, in the framework of directed interaction topology, the Laplacian matrix associated with the interaction topology is not symmetric and further, the system dynamics corresponding to time interval $[t_{i_j}, t_{i_{j+1}})$, across which the union of the interaction topology has a directed spanning tree, may be different for different j. In view of such, it is rather difficult, if not impossible, to construct a feasible Lyapunov function to deal with the convergence analysis under the repeatedly jointly rooted interaction topology, not to mention the estimation of a convergence rate.

(b) Consensus control is investigated upon a very general setting. The weighting factors are allowed to change dynamically to model more practical dynamics and the interaction topology switches among an infinite set of weighted directed graphs as opposed to the finite undirected ones in Ni and Cheng (2010) and Wang et al. (2008). Moreover, interaction topology is only required to be repeatedly jointly rooted which, thus extends the assumption on the interaction topology concerning dynamic case in Ni and Cheng (2010) and Wang et al. (2008) to a more general setting although at the cost of the full-state coupling between agents, comes out with a more relaxed assumption on A than that in Ni and Cheng (2010) and Wang et al. (2008). More specifically, A is allowed to have exponentially unstable mode and an upper bound for such unstable mode and the convergence rate can be specified as well. To the best of our knowledge, for agents with linear system dynamics, very few results are reported to study such two quantities (i.e., maximum allowable bound for the unstable mode of A and the convergence rate) even under the fixed interaction topology. This paper is one of the very few attempts to consider both of the two points under relaxed switching interaction topologies.

The rest of the paper is organized as follows. Some useful matrix and graph theory notations as well as the distributed control algorithm are introduced in Section 2. Section 3 presents the main result—key conditions under which the consensus control can be realized. In Section 4, we present some simulation examples to demonstrate the effectiveness and advantages of our theoretical finding. Finally, the main contributions are summarized in Section 5.

2. Preliminaries

2.1. Graph and matrix notation

The following notations will be used throughout the paper. Let $|\cdot|$ denote the 2-norm of a vector, and $\|\cdot\|_F$ denote the Frobenius norm of a matrix. Identity matrix in $\mathbb{R}^{p\times p}$ is denoted by I_p and zero matrix in $\mathbb{R}^{p\times q}$ by $0_{p\times q}$. Let $\mathbf{1}_N$ (0_N) be the column vector with all entries equal to 1 (0), $\mathrm{Re}(\lambda)$ be the real part of complex number λ. When the subscripts m and n are dropped, the dimensions of these vectors and matrices are assumed to be compatible with the context. Denote by $M > 0$ ($M < 0$) that M is symmetric positive (negative) definite. Denote by $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ the block diagonal matrix with its ith main diagonal matrix being a square matrix A_i, $i = 1, \ldots, n$. A matrix M is said to be nonnegative if all its entries are nonnegative. Let $\otimes$ be the *Kronecker product* and the Kronecker product has the following properties Horn and Johnson (1991):

Lemma 1. *For matrices A, B, C, and D with compatible dimensions, one can get*

(1) $(\alpha A) \otimes B = A \otimes (\alpha B)$, *where α is a constant;*
(2) $(A + B) \otimes C = A \otimes C + B \otimes C$;
(3) $(A \otimes B)(C \otimes D) = (AC \otimes BD)$;
(4) $(A \otimes B)^{\mathrm{T}} = A^{\mathrm{T}} \otimes B^{\mathrm{T}}$;
(5) $e^{I\otimes A} = I \otimes e^{A}$ *and* $e^{B\otimes I} = e^{B} \otimes I$;
(6) $\frac{d(A(t)\otimes B(t))}{dt} = \frac{dA(t)}{dt} \otimes B(t) + A(t) \otimes \frac{dB(t)}{dt}$;
(7) $\|A \otimes B\|_F = \|A\|_F \cdot \|B\|_F$.

A nonnegative matrix M is said to be row stochastic if all its row sums are 1. Let S_d denote the set of N by N row-stochastic matrices with positive diagonal elements, and $S_d(\upsilon)$ denote the matrices