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Brief paper

Cluster synchronization in directed networks of partial-state coupled linear systems under pinning control ☆

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ABSTRACT

This paper investigates the cluster synchronization for network of linear systems via a generalized pinning control strategy which allows the network of each cluster to take relaxed topological structure. For the case with fixed topology, it is shown that a feasible feedback controller can be designed to achieve the given cluster synchronization pattern if the induced network topology of each cluster has a directed spanning tree and further compared to the couplings among different clusters, the couplings within the each cluster are sufficiently strong. An extra balanced condition is imposed on the network topology of each cluster to allow for the cluster synchronization under arbitrary switching network topologies. Such a balanced condition can be removed via the use of dwell-time technique. For all the cases, the lower bounds for such strengths of couplings within each cluster that secure the synchronization as well as cluster synchronization rate are explicitly specified. Finally, some illustrative examples are provided to demonstrate the effectiveness of the theoretical findings.

1. Introduction

The studies on the complete consensus/synchronization of networked multi-agent systems, with the aim to reach an agreement regarding a certain quantity of interest that depends on the state of all agents, has received much attention in the past decades, see, e.g., Li, Duan, Chen, and Huang (2010), Li and Zhang (2010), Lin, Francis, and Maggiore (2007), Ma and Zhang (2010), Scardovi and Sepulchre (2009), Shen, Wang, and Hung (2010), Su and Huang (2012), Xiao and Wang (2008), Yu, Chen, Wang, and Yang (2009) and Zhang and Tian (2009), just to name a few. However, a real-world complex network may be composed of interacting smaller subnetworks and such a network in general exhibits richer scenarios than just consensus or synchronization, for example, communities of natural oscillators are usually composed of interacting sub-populations (Winfree, 1980). The phenomenon of cluster synchronization is observed when an ensemble of oscillators splits into several subgroups, called clusters throughout the paper, of synchronized elements. To start with the pioneer work in understanding the mutual synchronization between different neighboring visual columns (Gray, König, Engel, & Singer, 1989), increasing attention has been paid to cluster consensus/synchronization during the past few decades (Belykh, Osipov, Petrov, Suykens, & Vandewalle, 2008; Pogromsky, Santoboni, & Nijmeijer, 2002; Wu, Zhou, & Chen, 2009; Xia & Cao, 2011; Yu & Wang, 2010). Cluster synchronization is considered to be of high relevance in engineering control, social and ecological science, see, e.g., Dolby and Grubb (1998) for a foraging group with mixed species in nature and Blondel, Hendrickx, and Tsitsiklis (2009) for the opinion dynamics models in social networks where agents with bounded confidence levels finally evolve into different clusters.

A scheme with the presence of negative couplings among different clusters is employed in Wu et al. (2009) to consider the clustering for full-state coupled identical oscillators via pinning control

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techniques, where the nodes within the same cluster asymptotically reach synchronization and there is no synchronization among nodes from different clusters by choosing appropriate pinning nodes. This scheme is later developed in Yu and Wang (2010) to investigate group consensus[2] for agents with integrator dynamics, where some sufficient conditions in terms of linear matrix inequalities to ensure the group consensus are proposed. This group consensus algorithm is later revisited in Xia and Cao (2011), where an algebraic condition of simpler form is presented to guarantee the group consensus. Moreover, it is pointed out that a thin set of initial states needs to be excluded to guarantee the cluster consensus.

As a separate issue, synchronization problem for complex networks of general linear systems which are partial-state coupled, has also spurred great interests in systems and control, motivated partly by its ability to describe a broader class of complicated models in real applications. Different from the integrator agents where it is couplings among the agents only that determine the behaviors of the agents, the synchronization for the network of linear systems depends on not only the couplings among the linear systems (shall be termed nodes hereafter) in the network but also the self-dynamics governing the evolution of each isolated node. This makes the synchronization for network of linear systems technically much more challenging than the case for integrator agents. Although there has been an extensive and increasing literature addressing the synchronization problems from different perspectives (Huijberts & Nijmeijer, 2000; Li et al., 2010; Ma & Zhang, 2010; Qin, Gao, & Zheng, 2014; Scardovi & Sepulchre, 2009; Su & Huang, 2012), very few results are reported in regard to the cluster synchronization in networks of partial-state coupled linear systems with an exception in Qin and Yu (2013a). A special coupling mode among clusters is proposed in Qin and Yu (2013a) to guarantee the achieving of cluster synchronization regardless of the magnitudes of the couplings among nodes. However, it is still unclear for general coupling mode, under what conditions can cluster synchronization be achieved. Further, can the synchronization speed be explicitly specified and what are the lower bounds on the coupling strengths among nodes within each cluster that are necessary to secure the synchronization?

With mainly the above inspirations, this paper focuses on the cluster synchronization problem for a group of partial-state coupled linear systems with general coupling mode. Both the cases with fixed topology and that with switching topologies are considered. Under fixed network topology, it is shown that a feasible feedback matrix based on Riccati inequality can be designed to achieve the cluster synchronization as long as the induced network topology of each pinned cluster has a directed spanning tree and further compared to the couplings among different clusters, the couplings within the each cluster are sufficiently strong. The idea is then extended to deal with switching topology cases: the setting allowing for arbitrary switching topologies is tackled by assuming further that the network topology satisfies a certain balanced condition; while dwell-time technique (Morse, 1996) is employed to tackle the general setting that the network topology is imposed with relaxed connectivity constraint. Different from the synchronization analysis in Xia and Cao (2011), Yu and Wang (2010), this paper not only performs the cluster synchronization analysis but also designs the lower bounds for the coupling strengths within clusters which are necessary to secure the cluster synchronization and specify the synchronization rate as well. It is worthwhile to remark here that although the pinning control technique is also employed to help achieve the given cluster pattern, this paper differs from the work in Liu and Chen (2011) and Wu et al. (2009) in mainly the following two points: (1) such linear systems in the network are partial-state coupled however the couplings of the nodes in Liu and Chen (2011) and Wu et al. (2009) are of full-state; (2) only fixed network topology is considered in Liu and Chen (2011), Wu et al. (2009) and further, restrictive assumptions are made on the network topology. Specifically, couplings are assumed to be symmetric in Wu et al. (2009) and although the couplings are relaxed directed in Liu and Chen (2011), it is still assumed that the network topology of each cluster is strongly connected. In contrast, the general pinning scheme (Qin, Zheng, & Gao, 2011) employed in this paper allows for the most general topological structure of each cluster under fixed topology.

The rest of the paper is organized as follows. A brief summary of some preliminaries on the problem to be considered is provided in Section 2. Sections 3 and 4 consider respectively the cases under fixed and switching topologies. Several simulation examples are provided in Section 5 to demonstrate the effectiveness of the theoretical findings. Finally, some conclusion remarks are made in Section 6.

Notations: Let $\|x\| = \sqrt{x^{\mathrm{T}}x}$ denote the Euclidean norm of a finite dimensional vector x. Denote by I_n the identity matrix and by $0_{n\times n}$ the zero matrix in $\mathbb{R}^{n\times n}$. Let $\mathbf{1}_m$ ($\mathbf{0}_m$) be the column vector with all entries equal to 1 (0). When the subscripts m and n are dropped, the dimensions of these vectors and matrices are assumed to be compatible with the context. Let $diag\{\Xi_1, \ldots, \Xi_p\}$ denote the block diagonal matrix with the ith main diagonal block being square matrix Ξ_i, $\lambda_{min}(M)$ and $\lambda_{max}(M)$ denote respectively the smallest and largest eigenvalues of symmetric matrix M.

2. Preliminaries

2.1. Graph theory notation

We first recall some graph theory notions in this subsection (Godsil & Doyle, 2001). Let $G = (\mathscr{V}, \varepsilon, \mathscr{A})$ be a weighted digraph of order N with a finite nonempty set of nodes $\mathscr{V} = \{1, 2, \ldots, N\}$, a set of edges $\varepsilon \subset \mathscr{V} \times \mathscr{V}$, and a weighted *adjacency matrix* $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where a_{ij} is the weight of the directed edge (j, i) satisfying $a_{ij} \neq 0$ if (j, i) is an edge in G and $a_{ij} = 0$ otherwise. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathscr{V}$.

The Laplacian matrix $\mathscr{L}(G) = L = [l_{ij}]$ of weighted digraph $G = (\mathscr{V}, \varepsilon, \mathscr{A})$ is defined by $l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} a_{ik} & j = i \\ -a_{ij} & j \neq i \end{cases}$. A digraph G is called balanced if and only if all of its nodes are balanced, i.e., $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$, $\forall i \in \mathscr{V}$, or equivalently $\mathbf{1}^{\mathrm{T}}\mathscr{L}(G) = 0$ (Godsil & Doyle, 2001). A digraph *has a directed spanning tree* if there exists at least one node, called the root, having a directed path to all of the other nodes.

As is standard $\{\mathscr{V}_1, \ldots, \mathscr{V}_q\}$ is called a partition of the set $\mathscr{V} = \{1, 2, \ldots, N\}$ if $\mathscr{V}_\ell \neq \varnothing$, $\cup_{\ell=1}^{q} \mathscr{V}_\ell = \mathscr{V}$, and $\mathscr{V}_\ell \cap \mathscr{V}_k = \varnothing$ for $\ell \neq k$, $\ell, k = 1, \ldots, q$, but in this work the subsets of the partition are known as clusters. For $i \in \mathscr{V}$, let $\bar{i}$ denote the subscript of the subset to which the integer i belongs, i.e. $i \in \mathscr{V}_{\bar{i}}$. We say that agents i and j are in the same cluster if $\bar{i} = \bar{j}$. Let G_ℓ denote the underlying topology of cluster $\mathscr{V}_\ell$, $\ell = 1, \ldots, q$, i.e., $\mathscr{V}_\ell = \mathscr{V}(G_\ell)$.

2.2. Problem formulation

Consider a group of diffusively coupled linear systems which are modeled as follows:

$$\dot{x}_i(t) = Ax_i(t) + BK\Big[\sum_{j=1}^{N} c_{ij}a_{ij}\big(x_j(t) - x_i(t)\big) + u_i(t)\Big], \tag{1}$$

[2] Group consensus here means that all the nodes within the same cluster reach consensus while there may or may not be consensus among nodes from different clusters, depending on the choice of the initial conditions. This is slightly different from the cluster synchronization/consensus considered in Qin and Yu (2013a), Xia and Cao (2011), and Wu et al. (2009), where in addition to the group consensus requirement, there is no consensus among nodes from different clusters.