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Brief paper

Discerning controllers for switching linear systems: Existence and genericity[☆]

Marco Baglietto[a], Giorgio Battistelli[b], Pietro Tesi[c,1]

[a] Department of Informatics, Bioengineering, Robotics, and Systems Engineering -DIBRIS, University of Genoa, Italy

[b] Dipartimento di Ingegneria dell'Informazione -DINFO, University of Florence, Italy

[c] ITM, Faculty of Mathematics and Natural Sciences, University of Groningen, The Netherlands

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ABSTRACT

This paper presents recent developments in the study of non-autonomous switching linear systems. For such systems, we address the issue of how to systematically design linear controllers allowing the active process mode to be observable from closed-loop data. The results are stated formally by introducing the notion of *discerning controllers*. Both existence and genericity problems are discussed. It is finally shown how a given family of discerning controllers can be implemented as a single *hybrid* system which preserves the discerning capability of the original controllers. Examples are discussed to substantiate the analysis.

1. Introduction

Recent years have seen intensive research into systems that are characterized by the interaction of continuous and discrete dynamics, commonly referred to as hybrid systems (Liberzon, 2003). Switching linear systems represent a special class of hybrid systems, namely those systems composed of several linear subsystems (*modes*) and a switching signal that specifies the active subsystem at each instant of time. Switching systems can be classified in many ways: an important distinction is between *autonomous* and *non-autonomous* systems. This paper deals with the latter case, and focuses on the feedback configuration of Fig. 1. The process to be controlled is assumed to be modeled by a SISO switching linear system $\mathscr{P}_\rho$, namely a process composed by a family

$$\mathscr{P} := \{\mathscr{P}_i;\ i = 1, 2, \ldots, N\}$$

of linear time-invariant (LTI) subsystems $\mathscr{P}_i$ plus a signal ρ taking values in $\mathcal{N} := \{1, 2, \ldots, N\}$ that determines which subsystem is active at each time instant. It is supposed that the switching signal ρ is *neither controllable nor available for measurements*. The problem under consideration is that of designing a family

$$\mathscr{C} := \{\mathcal{C}_j;\ j = 1, 2, \ldots, M\}$$

of LTI controllers satisfying the following two requirements: *stabilization* and *global discernability*.

The *stabilization* requirement amounts to asking that, for each $\mathscr{P}_i \in \mathscr{P}$, there exists at least one candidate controller $\mathcal{C}_j \in \mathscr{C}$ able to internally stabilize $\mathscr{P}_i$. This requirement naturally arises when a single LTI controller cannot be found that guarantees stability and performance over the entire range of process operating modes, *e.g.* when two process modes are not simultaneously stabilizable (Doyle & Francis, 1990). Under such circumstances, a viable solution consists in designing a family of controllers so that, in each configuration, the process performs satisfactorily when fed-back by at least one of the available controllers.

The *global discernability* requirement amounts to asking that each element of $\mathscr{C}$ should allow indentifiability of the active process mode based on its input–output records. Although a precise definition of global discernability is given later on in Section 2, such a requirement can be intuitively motivated as follows. As an example, when the process switching signal is neither controllable nor accessible for measurements, the controller selection must rely on some type of logic-based unit that determines which candidate controller should be placed into feedback at each time instant (Morse, 1995). The simplest way to do this is to *estimate* the active mode of the process based on its input–output records, and select the candidate controller designed for the model that best

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E-mail addresses: marco.baglietto@unige.it (M. Baglietto), giorgio.battistelli@unifi.it (G. Battistelli), p.tesi@rug.nl (P. Tesi).

[1] Tel.: +31 050 3637649.

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fits the available data, which is basically an extension of the principle of *certainty equivalence* from tuning to switching (Hespanha & Morse, 1999; Morse, 1995). The discernability requirement goes precisely into this direction, addressing the problem of determining the conditions under which the process mode can be *exactly* reconstructed in the absence of process disturbances and measurement noises. For this reason, this is sometimes also referred to as the *mode-observability* requirement (Babaali & Egerstedt, 2004; Babaali & Pappas, 2005; Vidal, Chiuso, Soatto, & Sastry, 2003).

Recent results (Baglietto, Battistelli, & Tesi, 2013; Battistelli, 2013) have shown that control schemes based on the above two requirements provide quite strong stability properties to the closed-loop system, namely exponential input-to-state stability. A question therefore arises about the possibility to *satisfy these requirements at the design stage*. Preliminary results in this direction have been reported in Battistelli (2013). However, the results there provided are restricted to the zero-regulation case, and account neither for fixed structure controllers nor for implementation issues. Motivated by these considerations, the aim of this paper is to provide a general and in-depth treatment of the topic, taking both theoretical and practical aspects into account. In this respect, the contribution of the present paper is threefold: (i) we consider the framework of *generalized tracking* (Isidori, Marconi, & Serrani, 2003), providing necessary and sufficient conditions for a feedback controller to jointly satisfy regulation/tracking and discernability requirements; (ii) we provide an explicit characterization of discerning controllers within any given prescribed *parametric* controller family, which is of relevance when it is desired to adopt controllers with a prescribed structure, *e.g.* internal-model-based controllers. In this respect, the results show that whenever discerning controllers exist, then they exist *generically*, *i.e.* for almost all choices of their freely assignable parameters; (iii) finally, we show how a given family of controllers satisfying the regulation/tracking and discernability requirements can be implemented, rather than in the multi-controller configuration of Fig. 1, as a single *hybrid* dynamical system which preserves all the properties of the original controllers.

A final point is worth mentioning. The problem of discernability is not new in the literature on switching linear systems. However, this problem has been mainly addressed in connection with autonomous systems. For non-autonomous systems, the available results are in fact restricted to the case where no design objectives other than discernability are present (Babaali & Egerstedt, 2004; Babaali & Pappas, 2005; Baglietto, Battistelli, & Scardovi, 2007, 2009). As such, they are mainly oriented toward establishing connections between input selection and discernability, in close analogy with the problem of input selection for parameter identification (Grewal & Glover, 1976). By addressing the question of discernability in connection with standard feedback control design, the present paper then provides a more practical and effective methodology to achieve mode-observability in switching linear systems.

The remainder of the paper is as follows. In Section 2, we describe the framework under consideration. Section 3 provides the main results of the paper, addressing the question of discernability and its genericity properties in connection with standard feedback control design. In Section 4, implementation issues are discussed in details. Finally, an example is given in Section 5 and some concluding remarks are provided in Section 6.

Unless otherwise stated, all the proofs are reported in the Appendix.

2. Framework and objectives

Consider a process described by the switching linear system

$$\begin{cases} \dot{x} = A_\rho \, x + b_\rho \, u \\ y = c_\rho \, x \end{cases} \tag{1}$$

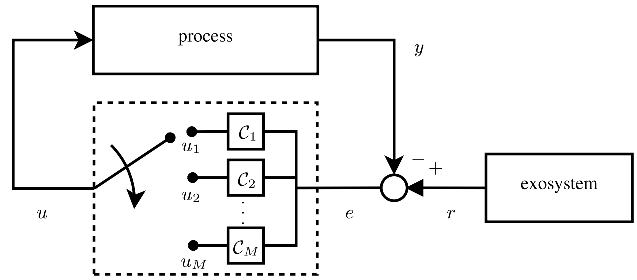

Fig. 1. Feedback loop with multi-controller architecture.

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output and $\rho : \mathbb{R}_+ \mapsto \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching signal, *i.e.* the (right continuous) signal which identifies the index of the active system at each instant of time, assumed to be *unknown*. A_i, b_i, and c_i, $i \in \mathcal{N}$, are matrices of appropriate dimensions. In the sequel, we shall denote by $\mathcal{P}_i$ the linear time-invariant (LTI) system with state-space representation $\{A_i, b_i, c_i\}$.

Along with the process, consider also a prescribed reference signal r. The latter is assumed to be generated by an LTI system $\mathcal{E}$ (hereafter referred to as the *exosystem*) of the form

$$\begin{cases} \dot{p} = E \, p \\ r = l \, p \end{cases} \tag{2}$$

where $p \in \mathbb{R}^{n_p}$ is the state. Without loss of generality, the pair (l, E) is supposed to be observable. This implies that $\mathcal{E}$ admits the polynomial description (Antsaklis & Michel, 2006, Section 7)

$$\Delta(q) r = 0 \tag{3}$$

where $q = d/dt$ and $\Delta(s) = \det(sI - E)$.

The problem under consideration is as follows. Assuming that ρ is *neither controllable nor available for measurements*, the aim is to design a family $\mathscr{C} := \{\mathcal{C}_j, \ j \in \mathcal{M}\}$, $\mathcal{M} := \{1, 2, \ldots, M\}$, of LTI controllers satisfying the following two requirements:

O1. *global discernability*;
O2. *stabilization*.

As for the *stabilization* requirement, we ask that for each $\mathcal{P}_i \in \mathscr{P}$, there exists at least one candidate controller $\mathcal{C}_j \in \mathscr{C}$ able to internally stabilize $\mathcal{P}_i$. In the following, we will also consider the situation where each controller should guarantee *offset-free* tracking for the corresponding process mode.[2] The reason for not explicitly regarding offset-free tracking as a design specification is for taking into account situations where it is desired to keep the controller order at a moderate value and to achieve a bound on the steady-state error rather than perfect tracking.

The *global discernability* specification amounts to asking that each element of $\mathscr{C}$ allows indentifiability of any possible active process mode from the plant input/output data

$$z = \mathrm{col}(u, y)$$

where *col* stands for column vector. To make this concept precise, some background material is needed. Let $e := r - y$ denote the tracking error, and let the dynamics of the jth controller $\mathcal{C}_j$ belonging to $\mathscr{C}$ be expressed as

$$\begin{cases} \dot{\xi}_j = F_j \, \xi_j + g_j \, e \\ u_j = h_j \, \xi_j + k_j \, e \end{cases} \tag{4}$$

[2] Here, by offset-free tracking we mean that, given an internally stable feedback interconnection $(\mathcal{P}_i/\mathcal{C}_j)$ with $i \in \mathcal{N}$ and $j \in \mathcal{M}$, then, for all initial conditions of $\mathcal{P}_i$, $\mathcal{C}_j$ and $\mathcal{E}$, the output of $\mathcal{P}_i$ converges to r as $t \to \infty$.