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Nonlinear decentralized control of large-scale systems with strong interconnections☆

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ABSTRACT

This paper addresses the problem of nonlinear decentralized state feedback stabilization for a class of large-scale systems with strong interconnections. The interconnections and their bounds are general functions of all the states. By developing a new recursive design method, a decentralized state feedback controller is successfully constructed for the large-scale system. The novelty of the proposed method is that a Lyapunov function in an appropriate product integral form is introduced at each step so that the recursive design can be carried out. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Nowadays, systems such as aircraft, networks, or air traffic control are becoming more and more complex. Control of large-scale interconnected systems has therefore received considerable attention over the past decades. To deal with the difficulties of dimensionality, information structure constraints and uncertainties, it is an efficient and effective way to formulate control laws using locally available states of the subsystems. For this reason, many works have been devoted to nonlinear decentralized control of large-scale systems; see, for instance, Guo, Jiang, and Hill (1999), Ioannou (1986), Jain and Khorrami (1997a,b), Jiang (2000), Jiang, Repperger, and Hill (2001), Wen (1994) and the references therein.

In Wen (1994), backstepping technique was employed for a class of large-scale systems with arbitrary relative degree. Since then, integrator backstepping was widely applied to nonlinear decentralized control of large-scale systems. In Jain and Khorrami (1997a) and Jain and Khorrami (1997b), decentralized controllers were constructed by utilizing the recursive method and the nonlinear interconnections were bounded by polynomial functions. These polynomial-type growth conditions were removed in Guo et al. (1999), Jiang (2000), Jiang et al. (2001). The reference Liu and Li (2002) considered a class of large-scale systems with unmodeled dynamics and Krishnamurthy and Khorrami (2003) further investigated large-scale systems in generalized output-feedback canonical form. Many related results (Guo, 2004; Liu & Huang, 2001; Liu, Jiang, & Hill, 2012; Wang, Khorrami, & Jiang, 2000; Ye, 2011; Zhou & Wen, 2008) were also reported on nonlinear decentralized control. However, these previous works usually considered weakly coupled subsystems, that is, the nonlinear interconnections and their bounding functions for each subsystem only contain its own states and the outputs of other subsystems.

Relative to weakly coupled subsystems, by strongly coupled subsystems, i.e., by subsystems with strong interconnections, we mean that the nonlinear interconnections and their bounding functions for each subsystem depend on all the states of the overall system. Decentralized design for strongly coupled subsystems was studied in Bakule (2008), Ghosh, Das, and Ray (2009), Stankovic and Siljak (2009) but the uncertainties have linear bounds and most of the works were based on LMI approach. In Dashkovskiy and Pavlichkov (2012), the authors considered a very general class of large-scale nonlinear systems and addressed the problem of input-to-state stabilization, but the proposed controller was not a decentralized one. To the best of our knowledge, no result has yet been reported about nonlinear decentralized control for strongly coupled subsystems.

The main contribution of this paper is to present, for the first time, a nonlinear decentralized design result for strongly coupled

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subsystems. Specifically, we consider the following class of large-scale nonlinear systems:

$$\begin{aligned}
\dot{x}_{i1} &= x_{i2} + f_{i1}(t, x_{11}, \ldots, x_{N1}),\\
&\vdots\\
\dot{x}_{in-1} &= x_{in} + f_{in-1}(t, x_{11}, \ldots, x_{1n-1}, \ldots, x_{N1}, \ldots, x_{Nn-1}),\\
\dot{x}_{in} &= u_i + f_{in}(t, x_{11}, \ldots, x_{1n}, \ldots, x_{N1}, \ldots, x_{Nn}),\\
y_i &= x_{i1},
\end{aligned} \tag{1}$$

where $x_i = [x_{i1}, \ldots, x_{in}]^T \in R^n$, $1 \le i \le N$, are the state vectors, $u_i \in R$ is the input of the ith subsystem, and the uncertain continuous functions $f_{ij}(t, x_{11}, \ldots, x_{1j}, \ldots, x_{N1}, \ldots, x_{Nj})$, $1 \le j \le n$, are locally Lipschitz in $(x_{11}, \ldots, x_{1j}, \ldots, x_{N1}, \ldots, x_{Nj})$ uniformly in t. The following assumption is made for the system.

A.1: The nonlinearities f_{ij} satisfy

$$|f_{ij}| \le \sum_{k=1}^{N} \gamma_{ijk}(x_{k1}, \ldots, x_{kj})(|x_{k1}| + \cdots + |x_{kj}|) \tag{2}$$

for known smooth functions $\gamma_{ijk}(x_{k1}, \ldots, x_{kj}) \ge 0$.

Under assumption A.1, it is shown that the interconnections can be bounded by nonlinear functions containing all the states of the overall system and therefore, the subsystems are strongly coupled. Since the states of each subsystem have the triangular structure at all the subsystems, (1) is in a generalized strict feedback form. When employing the conventional backstepping technique, a circular problem will occur such that the effects of the strong interconnections cannot be completely eliminated no matter how the virtual control signals and the actual control are chosen. With the quadratic Lyapunov function no longer effective, the conventional backstepping technique cannot be applied to the decentralized state feedback stabilization of the strongly coupled subsystems. Therefore, it is a challenging issue to develop a new recursive method for the large-scale system (1) satisfying assumption A.1. Furthermore, it is interesting to find an appropriate Lyapunov function for the controller design and stability analysis, which may lead to a feasible way for control of the strongly coupled subsystems.

The control objective is to design a decentralized state feedback controller such that the equilibrium state $x_e = 0$ of the closed loop system is globally asymptotically stable. Inspired by the idea of the changing supply functions (Sontag & Teel, 1995), a new storage function is introduced as the Lyapunov candidate at each step. The novelty is that the storage functions are in an appropriate product integral form and properly chosen such that the design procedure can be proceeded. Moreover, the virtual controllers are selected by taking the advantage of feedback domination design. Then, a new recursive method is successfully developed for the nonlinear decentralized stabilization problem. A numerical example is used to illustrate the effectiveness of the proposed method.

2. Main results

First of all, we introduce the variable separation technique (Lin & Qian, 2002) for the controller design and stability analysis.

Lemma 1 (*Lin & Qian, 2002*)**.** *For any real-valued continuous function $a(x, y)$, there are smooth functions $a_1(x) \ge 0$ and $a_2(y) \ge 0$, such that*

$$|a(x, y)| \le a_1(x)\, a_2(y). \tag{3}$$

Remark 1. With Lemma 1, the property of smooth function and Young's inequality, it is not difficult to prove that for any smooth function $g(x_1, \ldots, x_n)$ and positive integer κ, there are nonnegative smooth nondecreasing functions $g_i(\cdot) : \mathbb{R}^+ \to \mathbb{R}$, $1 \le i \le n$, such that

$$g(x_1, \ldots, x_n) \sum_{i=1}^{n} |x_i|^\kappa \le \sum_{i=1}^{n} g_i(|x_i|)\, |x_i|^\kappa. \tag{4}$$

It should be pointed out that the inequality (4) plays an important role in dealing with the system nonlinearities and can be easily worked out for many nonlinear functions.

Now, under assumption A.1, a new recursive method will be presented for the decentralized state feedback controller design.
Step 1: Define $z_{i1} = x_{i1}$. By viewing (1), the derivative of z_{i1} is expressed as

$$\dot{z}_{i1} = x_{i2} + f_{i1}(t, x_{11}, \ldots, x_{N1}). \tag{5}$$

Let

$$z_{i2} = x_{i2} - \alpha_{i1}(z_{i1}), \quad \alpha_{i1}(z_{i1}) = -\beta_{i1}(z_{i1})\, z_{i1}, \tag{6}$$

where the smooth design function $\beta_{i1}(z_{i1}) > 0$. Consider the Lyapunov function

$$V_{i1} = \frac{1}{2} z_{i1}^2. \tag{7}$$

Using (2), (4) and (6), the derivative of V_{i1} along the solutions of (5) is computed as

$$\dot{V}_{i1} \le -\beta_{i1}(z_{i1})\, z_{i1}^2 + z_{i2}^2 + \sum_{k=1}^{N} \mu_{i1k}^1(|z_{k1}|)\, z_{k1}^2 \tag{8}$$

for nonnegative smooth nondecreasing functions $\mu_{i1k}^1(|z_{k1}|)$, $1 \le k \le N$. Then, we further consider the Lyapunov function

$$V_{i1}^* = \int_0^{V_{i1}} q_1(s) \cdots q_{n-1}(s)\, ds, \tag{9}$$

where $q_j(s) : \mathbb{R}^+ \to [1, +\infty)$, $1 \le j \le n-1$, are smooth nondecreasing functions to be determined. It can be seen that V_{i1}^* is in a new product integral form, which makes it possible to eliminate the effects of the strong interconnections. From (8), the time derivative of V_{i1}^* satisfies

$$\begin{aligned}
\dot{V}_{i1}^* \le\ & q_1\left(\frac{1}{2} z_{i1}^2\right) \cdots q_{n-1}\left(\frac{1}{2} z_{i1}^2\right)\\
& \times \left[-\beta_{i1}(z_{i1})\, z_{i1}^2 + z_{i2}^2 + \sum_{k=1}^{N} \mu_{i1k}^1(|z_{k1}|)\, z_{k1}^2\right].
\end{aligned} \tag{10}$$

Then, consider the following two cases.

(i) $|z_{i1}| \ge |z_{i2}|$. From (10), it can be readily checked that

$$\begin{aligned}
\dot{V}_{i1}^* \le\ & q_1\left(\frac{1}{2} z_{i1}^2\right) \cdots q_{n-1}\left(\frac{1}{2} z_{i1}^2\right)\\
& \times \left[-\beta_{i1}(z_{i1})\, z_{i1}^2 + z_{i1}^2 + \sum_{k=1}^{N} \mu_{i1k}^1(|z_{k1}|)\, z_{k1}^2\right].
\end{aligned} \tag{11}$$

(ii) $|z_{i1}| \le |z_{i2}|$. In this case, we have

$$\begin{aligned}
\dot{V}_{i1}^* \le\ & q_1\left(\frac{1}{2} z_{i1}^2\right) \cdots q_{n-1}\left(\frac{1}{2} z_{i1}^2\right)\\
& \times \left[-\beta_{i1}(z_{i1})\, z_{i1}^2 + \sum_{k=1}^{N} \mu_{i1k}^1(|z_{k1}|)\, z_{k1}^2\right]\\
& + q_1\left(\frac{1}{2} z_{i2}^2\right) \cdots q_{n-1}\left(\frac{1}{2} z_{i2}^2\right) z_{i2}^2.
\end{aligned} \tag{12}$$