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Vector field radial basis function approximation☆

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ABSTRACT

Vector field simplification aims to reduce the complexity of the flow by removing features according to their relevance and importance. Our goal is to preserve only the important critical points in the vector field and thus simplify the vector field for the visualization purposes. We use Radial Basis Functions (RBF) approximation with Lagrange multipliers for vector field approximation. The proposed method was experimentally verified on synthetic and real weather forecast data sets. The results proved the quality of the proposed approximation method compared to other existing approaches. A significant contribution of the proposed method is an analytical form of the vector field which can be used in further processing.

1. Introduction

Interpolation and approximation are probably the most frequent operations used in computational techniques [1]. Several techniques have been developed for data interpolation and approximation, but they require some kind of data “ordering”, e.g. structured mesh, rectangular mesh, unstructured mesh etc. A typical example is a solution of partial differential equations (PDE), where derivatives are replaced by differences and rectangular or hexagonal meshes are used in the vast majority of cases. However, in many engineering problems, data are not ordered and they are scattered in k-dimensional space, in general. The k-dimensional space is sometimes not only spatial but also contains a time dimension or a dimension relating to age or temperature or other environmental conditions. Usually, in technical applications the scattered data are tessellated using triangulation, but this approach is quite prohibitive for the case of k-dimensional data interpolation or approximation because of the computational cost [2].

The technique for visualizing topological information in fluid flows is well known [3]. However, when the technique is used in complex and information-rich data sets, the result will be a cluttered image which is difficult to interpret. The paper [4] presents a simplification approach that removes pairs of critical points from the dataset, based on relevance measures. The approach does no grid changes since the whole method uses small local changes of the vector values defining the vector field. A simplification of vector field can be achieved by merging critical points within a prescribed radius into higher order critical points [5]. After building clusters containing the singularities to merge, the method generates a piecewise linear representation of the vector field in each cluster containing only one higher order singularity. Paper [6] presents a method to segment regions around a higher order critical point into areas of different 3*D* flow behavior. This method can be applied to any area of interest, e.g. around clusters of critical points. This can be used for a topological simplification tool by replacing the topological skeleton inside the area of interest. Combination of topological simplification technique and topology preserving compression for 2*D* vector fields is presented in [7]. A vector field is compressed in such way that its important topological features are preserved while its unimportant features are allowed to collapse and disappear. Dey et al. [8] present a Delaunay based algorithm for simplifying vector fields. The algorithm controls a local metric during removing vertices from Delaunay triangulation and maintains regions near critical points to prevent topological changes. The paper [9] uses a filtering technique based on the vorticity of the vector field to eliminate the less interesting critical points. The magnitude of the curl of the scalar field provides a basis to control the boundary thresholds as well as the number of critical points to include in the vector field. The paper [10] presents a technique for the visualization of multi-level topology in flow data sets. It provides the user with a mechanism to visualize the topology without excessive cluttering while maintaining the global structure of the flow. Skraba et al. [11,12] enable the pruning of sets of critical points

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according to a quantitative measure of their stability, that is, the minimum amount of vector field perturbation required to remove them. This leads to a hierarchical simplification scheme that encodes flow magnitude in its perturbation metric. A topological denoising technique based on a global energy optimization is proposed in [13], which allows the topology-controlled denoising of scalar fields. It allows processing small patches of the domain independently while still avoiding the introduction of new critical points. In the paper [14], they performed a numerical investigation of the differences between RBF global and local methods, in order to investigate the possible advantage of using local methods for the approximation of vector fields. The paper [15] presents a vector field approximation for two-dimensional vector fields that preserves their topology and significantly reduces the memory footprint. This approximation is based on a segmentation. The flow within each segmentation region is approximated by an affine linear function.

2. Vector field

Vector fields on surfaces are important objects, which appear frequently in scientific simulation in CFD (Computational Fluid Dynamics) [16,17] or modeling by FEM (Finite Element Method) [18,19]. To be visualized, such vector fields are usually linearly approximated for the sake of simplicity and performance considerations.

The vector field can be easily analyzed when having an approximation of the vector field near some location point. The important places to be analyzed are so called critical points [20]. Analyzing the vector field behavior near these points gives us the information about the characteristic of the vector field.

Critical points $\boldsymbol{x}_0$ of the vector field are points at which the magnitude of the vector vanishes

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{0}, \tag{1}$$

i.e. all components are equal to zero

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{2}$$

A critical point is said to be isolated, or simple, if the vector field is non vanishing in an open neighborhood around the critical point. Thus for all surrounding points $\boldsymbol{x}_\epsilon$ of the critical point $\boldsymbol{x}_0$ the equation (1) does not apply, i.e.

$$\frac{d\boldsymbol{x}_\epsilon}{dt} \neq \boldsymbol{0}, \tag{3}$$

At critical points, the direction of the field line is indeterminate, and they are the only points in the vector field where field lines can intersect (asymptotically). The terms singular point, null point, neutral point or equilibrium point are also frequently used to describe critical points.

These points are important because together with the nearby surrounding vectors, they have more information encoded in them than any such group in the vector field, regarding the total behavior of the field. The critical points are classified based on the vector field around these points, see Fig. 1.

3. Radial basis functions

Radial basis function (RBF) is a technique for scattered data interpolation [21] and approximation [22,23]. The RBF interpolation and approximation is computationally more expensive compared to interpolation and approximation methods that use an information about mesh connectivity, because input data are not ordered and there is no known relation between them, i.e. tessellation is not made. Although RBF has a higher computational cost, it can be used for d-dimensional problem solution in many applications, e.g. solution of partial differential equations [24,25], image reconstruction [26], neural networks [27–29], GIS systems [30,31], optics [32] etc. It should be noted that it does not require any triangulation or tessellation mesh in general. There is no need to know any connectivity of interpolation points, all points are tied up only with distances of each other. Using all these distances we can form the interpolation or approximation matrix, which will be shown later.

The RBF is a function whose value depends only on the distance from its center point. Due to the use of distance functions, the RBFs can be easily implemented to reconstruct the surface using scattered data in 2D, 3D or higher dimensional spaces. It should be noted that the RBF interpolation and approximation is not separable by dimension.

Radial function interpolants have a helpful property of being invariant under all Euclidean transformations, i.e. translations, rotations and reflections. It does not matter whether we first compute the RBF interpolation function and then apply a Euclidean transformation, or if we first transform all the data and then compute the radial function interpolants. This is a result of the fact that Euclidean transformations are characterized by orthonormal transformation matrices and are therefore two-norm invariant. Radial basis functions can be divided into two groups according to their influence. The first group are “global” RBFs [33]. Application of global RBFs usually leads to ill-conditioned system, especially in the case of large data sets with a large span [34,35].

The “local” RBFs were introduced in [36] as compactly supported RBF (CSRBF) and satisfy the following condition:

$$\begin{aligned} \varphi(r) &= (1-r)_+^q P(r) \\ &= \begin{cases} (1-r)^q P(r) & 0 \le r \le 1 \\ 0 & r > 1 \end{cases} \end{aligned} \tag{4}$$

where $P(r)$ is a polynomial function, r is the distance of two points and q is a parameter. The subscript in $(1-r)_+^q$ means:

$$(1-r)_+ = \begin{cases} (1-r) & (1-r) \ge 0 \\ 0 & (1-r) < 0 \end{cases} \tag{5}$$

3.1. Radial basis function approximation

RBF interpolation was originally introduced by Hardy [37] and is based on computing the distance of two points in any k-dimensional space. The interpolated value, and approximated value as well, is determined as (see [38]):

$$h(\boldsymbol{x}) = \sum_{j=1}^{M} \lambda_j \varphi(\|\boldsymbol{x} - \boldsymbol{\xi}_j\|) \tag{6}$$

where λ_j are weights of the RBFs, M is the number of the radial basis functions, φ is the radial basis function and $\boldsymbol{\xi}_j$ are centers of radial basis functions. For a given dataset of points with associated values, i.e. in the case of scalar values $\{\boldsymbol{x}_i, h_i\}_1^N$, where $N \gg M$, the following over-determined linear system of equations is obtained:

$$\begin{aligned} h_i = h(\boldsymbol{x}_i) &= \sum_{j=1}^{M} \lambda_j \varphi(\|\boldsymbol{x}_i - \boldsymbol{\xi}_j\|) \\ &\text{for } \forall\ i \in \{1, \dots, N\} \end{aligned} \tag{7}$$

where λ_j are weights to be computed; see Fig. 2 for a visual interpretation of (6) or (7) for a $2\frac{1}{2}D$ function. Point in $2\frac{1}{2}D$ is a $2D$ point associated with a scalar value. The same also applies to $3D$ point associated with a scalar value, thus $3\frac{1}{2}D$ point.

Eq. (7) can be rewritten in a matrix form as

$$\boldsymbol{A\lambda} = \boldsymbol{h}, \tag{8}$$

where $A_{ij} = \varphi(\|\boldsymbol{x}_i - \boldsymbol{\xi}_j\|)$ is the entry of the matrix in the ith row and jth column, the number of rows $N \gg M$, M is the number of unknown weights $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_M]^T$, i.e. a number of reference points, and $\boldsymbol{h} = [h_1, \dots, h_N]^T$ is a vector of values in the given points. The presented

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