

# Interval identification of structural parameters using interval overlap ratio and Monte Carlo simulation



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## ABSTRACT

In this paper, a new interval finite element (FE) model updating strategy is proposed for interval identification of structural parameters in the aspect of uncertainty propagation and uncertainty quantification. The accurate interval estimation of system responses can be efficiently obtained by application of Monte Carlo (MC) simulation combined with surrogate models. By means of the concept of interval length, a novel quantitative index named as interval overlap ratio (IOR) is constructed to characterize the agreement of interval distributions between analytical data and measured data. Two optimization problems are constructed and solved for estimating the nominal values and interval radii of uncertain structural parameters. Finally, the numerical and experimental case studies are given to illustrate the feasibility of the proposed method in the interval identification of structural parameters.

## 1. Introduction

In the past decades, especially in recent years, there has been ever-increasing interest in improving finite element predictions by use of experimental data as exact reference data, which is referred as FE model updating [1]. FE model updating is an inverse problem to identify and tune modeling parameters that leads to better predictions of the response behavior of an actual structure [2]. Deterministic model updating approaches [3–5] are now well-known and widely used in application to industrial-scale structures. In these approaches each of updating parameters is considered to be identified by minimizing the error between computed results and measured data from a single physical structure [6]. However, parameter uncertainty related to geometric dimensions and material properties exists in most real-world engineering structures. Hence, a model updating procedure involving uncertain analysis is of great importance and then the purpose of model updating becomes the estimation of ranges or distributions of parameters [7,8].

Stochastic model updating methods [9–19] have been developed for the treatment of parameter variability being caused by manufacturing tolerances of geometric dimensions, discreteness of material properties, etc. Fonseca et al. [11] proposed an updating algorithm based on maximum likelihood function. The MC based model updating procedure [9,10,12,13,18] is relatively easy to implement for identifying the parameter variability but it also requires the huge amount of

computational expense. Meanwhile, perturbation methods [15,16] have been successfully applied in stochastic model updating in which structural parameters and measured responses are presented as the summation of a deterministic part and a random variation.

However, an adequate probabilistic estimation always requires sufficient measurement information, which is often impractical in engineering practice [8,20]. In this circumstance, interval methods [21] are introduced as a useful alternative to quantify uncertain parameters. In the field of interval model updating in which the parameter uncertainties are described by interval numbers, some attempts [8,22,23] have been made to address an interval inverse problem. For solution of the interval model updating problem, Khodaparast et al. [8] first proposed the parameter vertex method which was valid only for particular parameterization of an FE model. Due to this drawback, they then presented the global optimization method for a general case based on the sensitivity analysis of the Kriging predictor. Fang et al. [22] proposed an interval response surface model (IRSM) for the interval model updating problem. The application of IRSM can simplify the establishment and implementation of the interval inverse problem, as this model helps to directly perform interval algorithms during the process of interval model updating. It should be noted that IRSM was constructed on the basis of a second-order polynomial model and was unable to consider interaction terms between updating parameters. Additionally, by use of the perturbation technique, Deng et al. [23] developed the two-step method which was used for updating the mean values and interval

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radii of uncertain parameters.

During the study and development of interval model updating, uncertain analysis is increasingly becoming one of the most critical techniques. The problem of uncertainty analysis can be generally categorized in two issues: uncertainty propagation and uncertainty quantification. Nowadays, the main interval methods for uncertainty propagation are the interval arithmetic [24], the perturbation method [25], the vertex method [8] and the MC simulation. The interval arithmetic including a set of operations often over-estimates the ranges of system responses since the correlation between the operands is neglected. The vertex method is valid only for particular cases with a monotonic relationship between the inputs and outputs. Although the perturbation method takes into account the calculation efficient, it depends on the initial value and uncertain levels of interval variables. Due to high accuracy and simple process, MC simulation is regarded as a suitable technique for uncertainty propagation of the input/output variables. However, the huge amount of calculation is an obstruction for the implementation of the direct MC simulation. On the other hand, the purpose of uncertainty quantification is to provide a quantitative metric that characterizes the agreement of interval ranges between analytical predictions and measured observations. So far, the common quantitative indexes for interval model updating are constructed on the basis of interval bounds [8,22], interval radii [23] or interval intersection [26,27]. However, these metrics do not wholly reflect the similarity and dissimilarity of the interval ranges between the analytical data and measured data.

As is discussed above, this paper presents a new strategy for interval model updating in the aspect of uncertainty propagation and uncertainty quantification. Firstly, uncertain parameters are selected together with interval distributions and are forward propagated through the model using MC analysis to provide the accurate interval ranges of the system responses. Simultaneously, to reduce the calculation amount of MC simulation, surrogate models are utilized in the process of updating parameter intervals. Secondly, the novel quantitative index named as IOR is constructed to characterize the agreement of interval distributions by use of the concept of interval length. Thirdly, two optimization equations are constructed and solved for estimating the nominal values and interval radii of uncertain structural parameters. Finally, the numerical and experimental case studies are given to illustrate the feasibility of the proposed method in the interval identification of structural parameters.

## 2. Basic theory for interval model updating

### 2.1. Description of uncertain parameters and uncertainty propagation

An uncertain mechanical system generally includes a set of input interval variables  $\mathbf{x}$  whose possible variations  $\delta\mathbf{x}$  around a nominal value  $\mathbf{x}^0$  create an uncertain model

$$\mathbf{x} = \mathbf{x}^0 + \delta\mathbf{x}, \delta\mathbf{x} \in [-\Delta\mathbf{x}, \Delta\mathbf{x}] \quad (1)$$

where  $\Delta\mathbf{x}$  denotes the interval radius vector of the uncertain parameters. Suppose there are totally  $p$  input variables and  $q$  output variables. These input variables might be different structural parameters such as material and geometrical properties, or aspects of modeling related to static and dynamic loads.

Due to high accuracy and simple process, MC simulation is regarded as a suitable technique for uncertainty propagation of the input/output variables. Generally, the uncertain parameters are assumed to follow certain interval forms that represent variation ranges of these input variables. A pair of input and output data points is generated after each FE-analysis as illustrated in Fig. 1. This process produces analytical predictions  $\mathbf{y}_s (s = 1, 2, \dots, N_1)$  ( $N_1$  is the number of FE outputs) and can be used for interval model updating when compared to the measured observations  $\mathbf{y}_t^m (t = 1, 2, \dots, N_2)$  ( $N_2$  is the number of experimental sampled points).

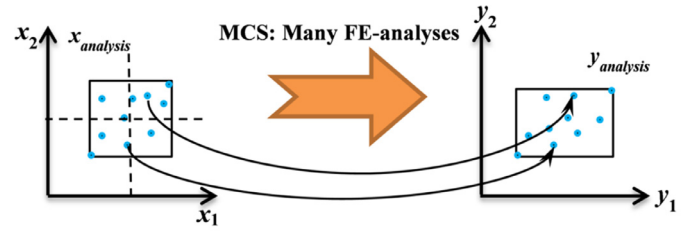


Fig. 1. MC simulation-based uncertainty propagation.

The nominal values of the analytical predictions and the measured observations, respectively, can be calculated as

$$\mathbf{y}^0 = \frac{1}{2} \left( \max_{1 \leq s \leq N_1} \mathbf{y}_s + \min_{1 \leq s \leq N_1} \mathbf{y}_s \right) \quad (2)$$

$$\mathbf{y}^{m0} = \frac{1}{2} \left( \max_{1 \leq t \leq N_2} \mathbf{y}_t^m + \min_{1 \leq t \leq N_2} \mathbf{y}_t^m \right) \quad (3)$$

And the interval radii of the analytical predictions and the measured observations, respectively, can be computed as

$$\Delta\mathbf{y} = \frac{1}{2} \left( \max_{1 \leq s \leq N_1} \mathbf{y}_s - \min_{1 \leq s \leq N_1} \mathbf{y}_s \right) \quad (4)$$

$$\Delta\mathbf{y}^m = \frac{1}{2} \left( \max_{1 \leq t \leq N_2} \mathbf{y}_t^m - \min_{1 \leq t \leq N_2} \mathbf{y}_t^m \right) \quad (5)$$

Then the variation intervals of the analytical predictions and the measured observations, respectively, can be described as

$$\mathbf{y}^l = [\underline{\mathbf{y}}, \overline{\mathbf{y}}] = [\mathbf{y}^0 - \Delta\mathbf{y}, \mathbf{y}^0 + \Delta\mathbf{y}] \quad (6)$$

$$\mathbf{y}^{ml} = [\underline{\mathbf{y}}^m, \overline{\mathbf{y}}^m] = [\mathbf{y}^{m0} - \Delta\mathbf{y}^m, \mathbf{y}^{m0} + \Delta\mathbf{y}^m] \quad (7)$$

where  $\overline{\cdot}$  and  $\underline{\cdot}$  denote the upper and lower bounds of a variable  $\cdot$ , respectively.

It should be noted that the huge amount of calculation is an obstruction for the implementation of the direct MC simulation. To make up the disadvantage of the direct use of MC simulation, surrogate models can be utilized with the purpose of representing the complex FE models for establishment of a cost-efficient updating problem. Thus, ones can choose a suitable type of surrogate models (such as the polynomial-based response surface model [22], the Kriging model [8], Radial Basis Function neural networks [23,28] and so on) according to the need of actual engineering problems. Since they are not the focus of this paper, discussions concerning the surrogate models are not elaborated here.

### 2.2. Interval overlap ratio

Quantitative metrics for characterizing the agreement of intervals between analytical responses and test responses are one of the most important techniques and directly affect the precision of an updating process. So far, the common quantitative indexes for interval model updating are constructed on the basis of interval bounds [8,22], interval radii [23] or interval intersection [26,27]. However, these indexes do not wholly reflect the degree of the similarity and dissimilarity of the interval distributions. Hence, a novel quantitative index named as IOR is constructed in this subsection.

For continuous intervals  $A = [\underline{a}, \overline{a}]$  and  $B = [\underline{b}, \overline{b}]$ , giving a definition of the length of the continuous intervals as follows

$$\text{len}(A) = \overline{a} - \underline{a}, \text{len}(B) = \overline{b} - \underline{b} \quad (8)$$

For an interval consisting of multiple continuous intervals, its length is the sum of each length of the continuous intervals. For example, the length of  $C = C_1 \cup C_2 \cup \dots \cup C_k$  are calculated as

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