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Research paper

Interval analysis method based on Legendre polynomial approximation for uncertain multibody systems

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ABSTRACT

An interval analysis method termed as Legendre interval model is presented to solve multibody dynamic systems with interval parameters. The implementation of Legendre interval model involves two steps. The first step is to approximate the original multibody model using Legendre polynomial approximation. Then take the interval parameters into consideration, and the Legendre interval model with interval parameters is obtained. The second step is to calculate the bounds of Legendre interval model using interval arithmetic or scanning. Legendre interval model using interval arithmetic, Legendre interval model using scanning, Chebyshev inclusion function method and scanning method are applied into typical multibody systems with interval parameters and compared in terms of efficiency and accuracy. The Legendre interval model using scanning shows high accuracy and efficiency.

1. Introduction

Multibody systems widely applied in articulated space structures, robotics, transportation vehicles, manufacturing equipment and biodynamic systems often operate with some uncertainties, which can results from variable parameters, from uncertain inputs, or from rapidly changing forces [1]. For realistic predictions of system performances, multibody systems often governed by index-3 differential algebraic equations (DAEs) [2] have to account for these uncertainties. Currently, three main kinds of methods, i.e., fuzzy, probabilistic and interval methods are used to formally assess the uncertainties. Beer et al. [3] reviewed a variety of uncertainty modeling approaches and their implementation techniques and applications.

The fuzzy-set theory initiated by Zadeh [4] is widely applied to the design and analysis of mechanical systems, such as fuzzy finite element analysis of dynamic systems [5], structural topology optimization [6] and vibration control [7,8], etc. The fuzzy theory is also used to solve the fuzzy differential equations [9], but most fuzzy-set methods are merely appropriate for linear differential equations or parameters with small intervals [10]. While fuzziness describes the ambiguity of the event, the fuzzy-set method is proper for the uncertainty due to vaguely defined system characteristics, imprecision of data, insufficient information and/or subjectivity of opinion or judgment [11]. One of the difficulties or challenges for fuzzy-set based treatments may be the definitions of membership functions which should be suitable for various types of uncertainties.

The probabilistic methods include the statistical and non-statistical methods. Statistical methods include the Monte Carlo method [12], Latin Hypercube sampling method [13], reliability-based methods [14] and response surface methods [15], etc. Monte Carlo method is used extensively in dynamic models although it is computationally expensive. Statistical information of system response is obtained from an ensemble of simulations with each member using a different set of uncertain parameters from the corresponding probability distribution. The polynomial chaos method is one of the most widely used non-statistical probabilistic methods. The basic idea of polynomial chaos method is that random processes can be approximated by sums of orthogonal polynomial chaos of random independent variables [1]. It has been successfully used to model uncertainty in many engineering applications, such as fluid mechanics [16] and multibody dynamics [17–19], etc. Li et al. [18] applied polynomial chaos method to study the tractive capability of off-road vehicles with uncertainties in suspension stiffness and damping, tire stiffness, terrain geometry and soil parameters. Kewlani et al. [19] applied polynomial chaos theory to vehicle dynamics under uncertainty, and shown that polynomial chaos was computationally more efficient than the standard Monte Carlo method.

While probability distributions for random variables or membership functions for fuzzy variables are not always available and obtaining such information is often quite expensive even if possible, interval

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uncertainty levels are often much easier to determine and/or estimate. Both the interval models [20] and ellipsoid models [21] which belong to convex models [22,23] are proper for modeling such systems with uncertain-but-bounded/interval parameters. For this reason, the interval method is experiencing popularity and has been widely studied and applied.

Taylor series based interval methods are widely used for static structural problems [24–27] and dynamic structural problems [28–33]. Qiu et al. [28] used the first-order Taylor expansion to estimate the dynamic response of nonlinear vibration systems with uncertainties. Qiu et al. [30] developed non-probabilistic interval analysis method for the dynamic analysis of structures based on parameter perturbation method. Wu et al. [31] used the first-order Taylor series to analyze the dynamic response of linear structural systems with interval parameters. Chen et al. [32] presented an interval optimization method for the dynamic response of structures with interval parameters. Liu et al. [33] studied the interval dynamic responses of vehicle-bridge interaction system with uncertain parameters. These methods require derivative information of the system so they are seldom used in complex dynamic systems. The main drawback of Taylor series based methods is the extremely overestimation of the real system responses unless the number of interval parameters is very small and the widths of the intervals are very small.

Recently, the Chebyshev inclusion function method [34] is proposed to solve multibody system dynamics with interval parameters. The Chebyshev inclusion function method is based on the best uniform approximation theory and can reduce the overestimation significantly in contrast to the Taylor inclusion function. Wang et al. [35] applied Chebyshev inclusion function method to study the dynamics of rigid-flexible multibody systems with a large number of interval parameters. In both [34] and [35], the scanning method is used to obtain the exact solutions in the interval analysis problems although it may cost much computation time when the number of interval variables is large.

The major goal of this paper is to present Legendre interval model based on Legendre polynomial approximation and apply it into complex uncertain multibody dynamic systems, e.g. a complete vehicle model. In Legendre interval model, constructing the approximated polynomials model is a key process, e.g. the Taylor inclusion function [28] and Chebyshev inclusion function [34] respectively use the Taylor expansion and Chebyshev polynomials expansion to approximate the original model.

This paper is organized as follows. In Section 2, basic knowledge about interval analysis is reviewed. In Section 3, the formulation of multibody system with interval parameters is derived. In Section 4, Legendre interval model is presented. In Section 5, numerical examples are shown. In Section 6, main conclusions are drawn.

2. Basic knowledge about interval analysis

An interval parameter represents a range of uncertainty. For each interval parameter, there are two numbers that represent the lower and upper bounds of the parameter. A real interval $[x]$ in real set R is defined as

$$[x] = [\underline{x}, \overline{x}] = \{\underline{x} < x < \overline{x} | x \in R\} \tag{1}$$

where $\underline{x}$ is the lower bound and $\overline{x}$ the upper bound. An interval $[x]$ can be expressed in the form of

$$[x] = x^C + [-1, 1]x^R \tag{2}$$

where x^C is the midpoint and x^R the radius.

Several interval variables form an interval vector. For instance, vector $[\mathbf{x}]$ includes m interval variables: $[x_1],[x_2],\cdots, [x_m]$ and each element has an interval bound, i.e. $[x_i] = [\underline{x}_i, \overline{x}_i]$, $i = 1, 2...m$. A function $f([\mathbf{x}])$ containing one or more interval variables $[\mathbf{x}]$ is called an interval function. Providing the bounds of $f([\mathbf{x}])$ is the major objective in interval analysis.

Many interval analysis methods have intrinsic overestimation caused by the intrinsic wrapping effect [36,37], meaning that the bounds of interval function are overestimated. For instance, the Taylor inclusion functions based on Taylor series can usually get large overestimation. Generally, the nth-order Taylor interval model is defined by [36]

$$f_T([x]) = f(x_c) + f'(x_c)[\Delta x] + \cdots + \frac{1}{n!} f^{(n)}(x_c)[\Delta x]^n + \frac{1}{(n+1)!}[f^{(n+1)}([x])][\Delta x]^{n+1}$$

where $x_c = (\underline{x} + \overline{x})/2$, $[\Delta x] = [-(\overline{x} - \underline{x})/2, +(\overline{x} - \underline{x})/2]$.

In many literatures [34,35], interval analysis methods are validated by the so-called scanning method which is based on a dense grid samples and can provide the exact bounds in interval analysis. The scanning method is widely used in interval analysis to provide exact solution. Take a problem with 3 interval variables for example. In the scanning method, a large number (e.g. 20) of sampling points are selected uniformly in each dimensional interval variable, so the total sampling points is 20^3. After calculate the system responses to all the sampling points the bounds of the system responses can be obtained.

Attention should be paid to the difference between Monte Carlo simulation (MSC) and scanning. MCS is based on the random sampling method, so it requires the probability distributions of uncertain variables. As a result, the MCS is more suitable for the probabilistic problems.

3. Problem formulation: multibody system with interval parameters

For dynamics of multibody systems, the equations of motion can be expressed by [2].

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^T(\mathbf{q}, t)\boldsymbol{\lambda} = \mathbf{Q}(t, \mathbf{q}, \dot{\mathbf{q}}) \\ \Phi(\mathbf{q}, t) = \mathbf{0} \end{cases} \tag{3}$$

where $\mathbf{q} \in R^n$ are the generalized coordinates, $\boldsymbol{\lambda} \in R^m$ are the Lagrange multipliers and t represents the time. The matrix $\mathbf{M}$ is the generalized mass matrix; $\mathbf{Q}$ represents the vector of generalized applied forces; $\boldsymbol{\Phi}$ represents the set of m holonomic constraints, i.e., position-level kinematic constraints.

Assume that the symbol $[\mathbf{x}]$ represents an interval vector and it may involve the uncertain factors resulting from geometry size, material properties and so on. The mass matrix $\mathbf{M}$, generalized applied forces $\mathbf{Q}$ and constraints $\boldsymbol{\Phi}$ should be expressed by interval functions:

$$\begin{aligned} \mathbf{M} &= \mathbf{M}(\mathbf{q},[\mathbf{x}]); \\ \boldsymbol{\Phi} &= \boldsymbol{\Phi}(\mathbf{q}, [\mathbf{x}], t); \\ \mathbf{Q} &= \mathbf{Q}(t, [\mathbf{x}], \mathbf{q}, \dot{\mathbf{q}}) \end{aligned} \tag{4}$$

Substituting Eq. (4) into Eq. (3), the dynamic equations of the multibody system with interval variables are finally formulated as

$$\begin{cases} \mathbf{M}(\mathbf{q},[\mathbf{x}])\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^T(\mathbf{q}, [\mathbf{x}], t)\boldsymbol{\lambda} = \mathbf{Q}(t, [\mathbf{x}], \mathbf{q}, \dot{\mathbf{q}}) \\ \Phi(\mathbf{q}, [\mathbf{x}], t) = \mathbf{0} \end{cases} \tag{5}$$

Many direct methods can be used to solve index-3 DAEs as shown in Eq. (5), such as the generalized-α method [38] and HHT-I3 method [39]. In this paper, HHT-I3 method is used to produce a numerical solution. When the interval variables are considered in the dynamic equations, the dynamic equations are discretized into the following algebraic equations