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Dynamic response of a Spur gear system with uncertain friction coefficient

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ABSTRACT

In this paper, we propose a method for taking into account uncertainties based on the projection on polynomial chaos. The proposed method is used to determine the dynamic response of a spur gear system with uncertainty associated to friction coefficient on the teeth contact. We developed a lumped dynamic model with 8dofs. Lagrange formalism is used to formulate the governing equation of motion of the model. The simulation results are obtained by the polynomial chaos method for dynamic analysis under uncertainty. The polynomial chaos results are compared with Monte Carlo simulations.

1. Introduction

The study and analysis of the dynamic behavior with nonlinear systems is a major interest in the industrial sector. Thus they allow overcoming the areas of instability and reducing vibration levels. Indeed, the negative consequences that may result from the instability of systems require designers to develop the most rigorous solution. This passes through a detailed study and analysis of the dynamic behavior of these systems before considering their actual implementation.

Several parametric studies have shown the great sensitivity of the dynamic behavior of gear systems. However, these parameters admit strong dispersions [1,2]. Therefore, it becomes necessary to take into account these uncertainties to ensure the robustness of the analysis. Also there are several studies in reliability for vibration structures taking into account the uncertainties [3–8].

The mechanisms of transmission by gear tooth contact are characterized by the presence of friction coefficient that affects the vibration and noise of these systems. Parameter estimation is an important problem, because many parameters simply cannot be measured physically with good accuracy, such us the friction coefficient, especially in real time application [9,10].

The coefficient of friction is a very important factor for designing, operating, and maintaining the gear transmission. Indeed, the accurate estimation of this coefficient is difficult due to the effects of various uncertain parameters, e.g., materials of gears, roughness and contact patch size, etc. However, the friction coefficient admits a strong dispersion [11]. Therefore, it becomes necessary to take into account these uncertainties in order to ensure the robustness of the analysis. A study of the nonlinear dynamic behavior will help to analyze stability and to predict the vibration levels according to the parameters variations.

Numerous methods have been used in recent years to quantify uncertainties in a variety of mechanical problems like the Monte Carlo simulation, the perturbation method and polynomial chaos method.

The Monte Carlo simulation is based on solving the deterministic problem multiple times for randomly chosen parameter values. MC simulation is a well-known technique in this field [12]. It can give the entire probability density function of any system variable, but it is often too costly since a great number of samples are required for reasonable accuracy. Parallel simulation [13] and proper orthogonal decomposition [14] are some solutions proposed to circumvent the computational difficulties of the MC method. Due to the slow convergence rate of standard Monte Carlo usually many samples are required. If solving the deterministic problem is already computationally intensive, the computational costs of a Monte Carlo simulation can become impractical.

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Nomenclature

Cm	motor torque (N.m)
Cf_{12} ,Cf_{21}	friction torque on the spur gear (21) and (21) (N.m)
$F_f(t)$	friction force
$F_f^0(t)$,$F_f^1(t)$	friction force of one and two pair of teeth in contact
k_1^x,k_1^y,k_2^x,k_2^y	traction-compression stiffness of the first and second bearing (N/m)
k_1^θ,k_2^θ	torsional stiffness of the shaft 1 and 2 (N.m/rad)
$k(t)$	gear mesh stiffness
kc	mean component of mesh stiffness
$kv(t)$	time mesh stiffness varying component
$k^0(t)$,$k^1(t)$	mesh stiffness of the one and two pair of teeth in contact
$\{Q\}$	generalized coordinate's vector
$r^b_{(1,2)}$, $r^b_{(2,1)}$	basic radius of the wheel (21) and (21)
$\theta_{(1,\,1)}$, $\theta_{(1,\,2)}$, $\theta_{(2,\,1)}$, $\theta_{(2,\,2)}$	angular displacement of the wheel (11),(12),(21) and (22) (°)
x_1,y_1,x_2,y_2	translational displacement of the first and second bearing (m)
μ	friction coefficient
μ_c	critical friction coefficient
δ	teeth deflection (m)
α	pressure angle (°)
ε^α	contact ratio
$[M_T]$,$[K_T]$	mass and stiffness matrices
$\{f_T\}$	external force vector
$[\tilde{M}_T]$, $[\tilde{K}_T]$	random mass and stiffness matrices
α_p	random variables
$\psi_m(\alpha_p)$	multidimensional orthogonal polynomials chaos
$[M_T]_0$, $[K_T]_0$	average of mass and stiffness matrices
$\langle..\rangle$	iner product defined by the mathematical expectation operator

A fast method for obtaining uncertainty information in terms of the first and second statistical moments is the perturbation method [15]. Here the statistical moments of the output are determined by expanding the stochastic quantities around their mean via a Taylor series expansion. The application of the perturbation method is, however, limited to small perturbations.

The polynomial chaos method [16] has proven to be a successful approach to solve uncertainty problems. It is an expansion of orthogonal polynomials in terms of random variables to approximate the full uncertainty distribution of the output. The original homogeneous polynomial chaos method is based on the homogeneous chaos theory of Wiener [17,18]. It employs Hermite polynomials and Gaussian random variables.

The exponential convergence of the polynomial chaos expansion has been extended to several other types of commonly used probability distributions in the generalized polynomial chaos by Xiu and Karniadakis [19]. In the generalized polynomial chaos several classical polynomials of the Askey scheme of hyper geometric orthogonal polynomials are employed, which are orthogonal with respect to other probability distributions such as the gamma and beta distributions. A multi-element version of the generalized polynomial chaos has been developed by Wan and Karniadakis [20] to improve the resolution of singularities in the random space and long-term integration. Generalized polynomial chaos and multi-element generalized polynomial chaos have been applied by Karniadakis and co-workers to for example flow problems and fluid–structure interaction problems in 21–24.

The capabilities of polynomial chaos have been tested in numerous applications, such as treating uncertainties in environmental and biological problems [25,26], in multibody dynamic systems [27,28], solving ordinary and partial differential equations [29,30]and parameter estimation [31–33].

This paper is an updated and revised version of the conference paper [3].In this paper we have proposed a method for taking into account uncertainties based on the projection on polynomial chaos. The proposed method is used to determine the dynamic response of a system with uncertainty associated to friction coefficient on the teeth contact.

The first contribution of the present paper is to study the stability of the system according to the friction coefficient values. The second contribution is to analyze the dynamic response of the system with friction coefficient. The third contribution is that the uncertainty of the gear friction system in the dynamic behavior study of one stage gear system is taken into account. The main objective is to investigate of the capabilities of the proposed method to determine the dynamic response of a spur gear system subject to uncertain friction coefficient. So, an eight degree of freedom system modelling the dynamic behavior of a spur gear system is considered. The modelling of a one stage spur gear system is presented in Section 2. The modelling of friction coefficient is presented in Section 3. In the next section, the theoretical basis of the polynomial chaos is presented. In Section 5, the equations of motion for the eight degrees of freedom are presented. Numerical results are presented in Section 6. Finally in Section 7, to conclude, some comments are made based on the study carried out in this paper.

2. One-stage spur gear system modelling

The global dynamic model of the one stage gear system in 2D is shown in Fig. 1. This model is composed of two blocks. Every block is supported by flexible bearing which the bending stiffness is k_1^x and the traction-compression stiffness is k_1^y for the first block, k_2^x and k_2^y for the second block.

The wheels (11) and (22) characterize respectively the drive and driven gears. The two shafts (1) and (2) admit some torsional stiffness k_1^θ and k_2^θ.

Angular displacements of every wheel are noticed by $\theta_{(1,\,1)}$, $\theta_{(1,\,2)}$, $\theta_{(2,\,1)}$ and $\theta_{(2,\,2)}$. Besides, the linear displacements of the bearing noted by x_1 and y_1 for the first block, x_2 and y_2 for the second block, are measured in the plan which is orthogonal to the wheels axis of rotation [34].

Fig. 2 defines a reference frame $(O, \overrightarrow{X}, \overrightarrow{Y})$ and the position of the wheels of the one stage gear system. α is the pressure angle of gearmesh contact.

The teeth deflection is expressed along the line of action, it can be written by:

$$\delta(t) = s^\alpha(x_1 - x_2) + c^\alpha(y_1 - y_2) + r^b_{(1,2)}\theta_{(1,2)} - r^b_{(2,1)}\theta_{(2,1)} \quad (1)$$

Where $s^\alpha = \sin(\alpha)$ and $c^\alpha = \cos(\alpha)$ and $r^b_{(1,2)}$, $r^b_{(2,1)}$ represent the base gears radius.

Generally the gear mesh stiffness variation k(t) is modeled by a sinusoid wave or by a square wave form. The later is the most representative of the real phenomenon and is represented on Fig. 3. The gear mesh stiffness variation can be decomposed in two components: an average component noted by kc, and a time variant