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A direct and fully general implementation of influence lines/surfaces in finite element software

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ABSTRACT

In the paper a general and direct method for implementation of influence lines in finite element software is provided. Generally influence lines are applied to identify the most critical location and combination of live loads in civil engineering structures. The proposed method is based on the Müller-Breslau principle and the basic idea is to equate discontinuous displacement fields with consistent nodal forces, thus obtaining influence functions only applying a single load case without changing the geometry or boundary conditions of the finite element model. Initially the method is developed by means of some illustrative beam problems, where the consistent nodal forces for angular, lateral and axial displacement discontinuities for a Bernoulli-Euler beam element are derived. Finally it is shown that the method is fully general and efficient in identifying the influence functions of generalized stresses in e.g. plates and shells.

1. Introduction

In the design of civil engineering structures, which are subjected to extensive live loads, it is of great concern to identify the load condition that causes the most critical structural response in an arbitrary design point of the structure. Where the location of live loads have to be identified in more complex structures, it is not an intuitive task. Each structural element has to be designed to carry the worst live load combination and here the influence lines efficiently identify where the live load has to be located, to cause the most critical structural response, e.g., the displacements, the sectional forces or stress states in a given point *i*. Currently none of the leading general finite element software packages, such as ANSYS, ABAQUS, ADINA, MSC Nastran have, to the authors knowledge, applications for constructing influence lines/surfaces. Software packages such as SAP2000 and Autodesk Robost Strutural analysis, which targets more directly civil engineering design problems, have subroutines capable of constructing influence lines or surfaces by means of unit load stepping. The aim of this paper is to facilitate a more direct and efficient method for obtaining these influence functions, without unit load stepping. The method can be implemented by any structural finite element software, commercial or not. The method of constructing influence functions is initially explained by beam problems, where the influence line is a function, $\delta^i(s)$, which states the influence in a point *i* for a unit force located at a point of coordinate *s*. The most utilized method for establishing influence functions in commercial finite element software is by means of a unit force stepped through the entire structure, while the influence is measured in node *i* of interest, see e.g., [1,2]. In Fig. 1 this is exemplified by constructing the influence line of the bending moment at the mid-section of a two span continuous beam. The continuous beam is subjected to a unit load P, which is stepped through the entire structure. For each increment of *s* the influence is measured at point *i* and is illustrated as the ordinate $\delta^i(s)$ for the unit load situated at distance *s*. In this case it is clearly seen that the worst location of concentrated live load P is in the middle of the left span, hence it gives the maximum positive influence, while live load situated in the right span causes a negative influence and thus a negative contribution to the moment in point *i*. The influence lines provides a sound interpretation of the critical position of the live load.

The paper in the succeeding sections will consider a more convenient calculation method for establishing influence lines, based on the Müller-Breslau principle, [4]. The principle is based on Betti's law of virtual work [5] and Maxwell's theorem of reciprocal deflections [6], which states that the virtual work done by the forces in a system 1 acting with the corresponding displacement field in system 2 is equal to the virtual work done by the forces in system 2 acting with the corresponding displacement field in system 1. The principle states that the influence line of a sectional force is constructed by the deflected shape of a structure where the constraint of a given point is released and subjected to a unit displacement or rotation discontinuity respectively. The principle is

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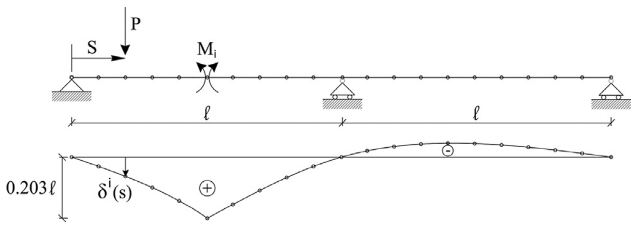

Fig. 1. Influence line for the bending moment at the middle section of the left span.

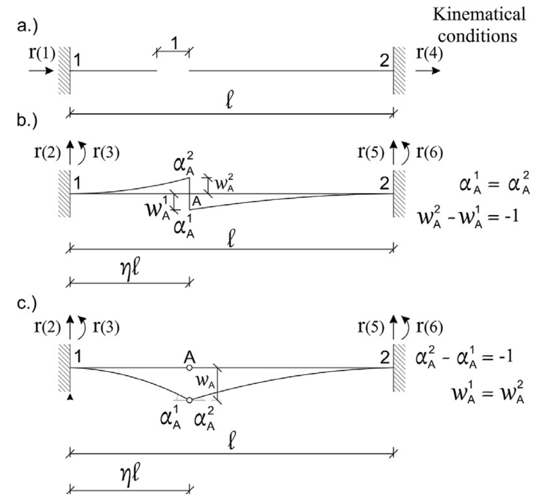

Fig. 3. Discontinuous displacement fields a.) Axial, b.) Transverse and c.) Angular.

sketched in Fig. 2, and it is seen that the only work performed, is between the internal moment of system 1 and the rotation discontinuity and additionally the external unit force of system 1 with the transverse deflection, $w(s)$, of system 2. Only reactions forces exists in system 2, which does not perform any work with the displacement field of system 1. According to Bettis law, the work performed by the two systems is expressed by:

$$1 \cdot w(s) + (-M) \cdot 1 = 0 \Rightarrow M = w(s) \quad (1)$$

Where $w(s)$ is the lateral displacement in the discontinuous displacement field of system 2, and $w(s)$ is thus the influence line of M. By means of the Müller-Breslau principle, the influence lines of sectional forces can be constructed for any given point i in the structure and the method is both suited for hand calculation and the finite element method, as shown in previous works [3,7,8,10]. In [7], the Müller-Breslau principle is explicitly modeled by means of the finite element method by releasing the degree of freedom corresponding to the given sectional force of concern and subject it to a kinematic restriction, as illustrated in Fig. 3. From a finite element modeling point of view, the method used in [7] inconveniently requires a new finite element model for each influence line of concern, because the boundary conditions have to be modified. [9] suggests a different approach, where the influence lines are computed by the adjoint method, which uses the influence coefficient, known from sensitivity analysis in structural optimization. The adjoint method is not directly suitable for implementation in commercial finite element software, because the influence coefficients are not a explicit result of the traditional finite element formulation, and consequently additional programming is needed. Therefore it is desirable to develop a more direct approach for computing influence lines, without the need of extra modification of the FE model or complex formulations of the finite element scheme.

2. New method – consistent forces

A more direct determination of influence lines in finite element software is achieved by representing the discontinuous displacement fields from the Müller-Breslau principle by means of consistent nodal forces. The method is accounted for in the following by means of simple 2D Bernoulli-Euler beam elements. In Fig. 3, the consistent nodal forces for axial and transverse translation and rotation discontinuity are sketched, and the detailed derivation of these consistent nodal forces for the 2D beam element can be seen in Appendix A. The direct determination method has also been proposed in [8], but this method does not take the local element deflection into account, hence special discretization of the FE model is required, to construct the correct influence line. Including the local contribution makes it possible to construct the entire influence line for a arbitrary design point, without changing the element discretization.

2.1. Calculation procedure

The procedure for constructing influence lines by consistent forces for a beam element is described in 5 steps. The method is initially exemplified by means of the three span beam in Fig. 2. The FE discretization is seen in Fig. 4, and only 3 elements are needed to construct the influence line of the bending moment at midspan in element 1. The influence line is determined by the following approach:

1. Select the element and locate the point i and select a stress component.
2. Determine the consistent nodal forces using Appendix A.
3. Subject the consistent nodal forces to the structure and determine the global displacement field w.
4. Determine the local displacement contribution w_l in the element with the discontinuity.
5. The influence function is determined as $w + w_l$.

Conducting step 1 to 3, the method provides the correct influence function outside the element, while the internal displacement is not correct internally, because it is not discontinuous. In the case of the 2D Bernoulli-Euler element, the local displacement effect of

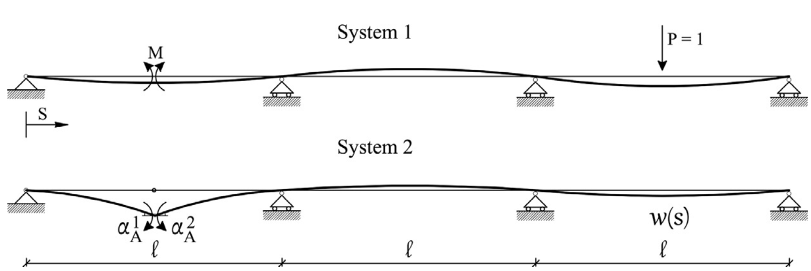

Fig. 2. Deflection figure for system 1 and 2.