Advances in Engineering Software xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Research paper

A critical analysis of expected-compliance model in volume-constrained robust topology optimization with normally distributed loading directions, using a minimax-compliance approach alternatively

A. Csébfalvi[a,*], J. Lógó[b]

[a] University of Pécs, Hungary
[b] Budapest University of Technology and Economics, Hungary

ARTICLE INFO

Keywords:
Robust topology optimization
Uncertain parameters
Uncertain loading direction
Expected compliance
Worst compliance
Minimax compliance model

ABSTRACT

Uncertainty is an important consideration in topology optimization to produce robust and reliable solutions. There are several possibilities to take into account the uncertainty in the topology optimization of continuum structures. In this paper, we assume that the only source of uncertainty is the variability of the applied load directions. Most models in this area apply parametric statistical tools to describe the directional uncertainty of the applied loads to produce robust structures which are insensitive to the directional uncertainty as much as possible. In the most popular parametric statistical approach the expected-compliance, based on the directional normality assumption, is used as the preferred measure of robustness. We will proof indirectly that this approach is far from the engineering practice and may give hardly interpretable or totally misleading results, which will be demonstrated by two carefully selected counter-examples. The counter-examples validate the fact that the expected-compliance, as a statistical abstraction based on more or less theoretical assumption about normality, not a general applicable measure of robustness. It will be shown, that the non-parametric and really robust volume-constrained worst-load-direction-oriented minimax-compliance model, used in this paper only as a proofing tool in a very simple form, is a viable alternative of the parametric expected-compliance model and its results and its problem solving process as a whole are very close to the engineering thinking. The worst-load-direction-oriented minimax-compliance-model provides expressive, rigorous, generally applicable, and objective information about the robustness. The parametric expected-compliance in itself as the preferred measure of robustness is unable to characterize the compliance variability, in contrast of the minimax approach which can be describe the compliance variability by a robust range-like measure computed very easily as the difference of the maximal- and minimal-compliance on the set of the feasible loading directions.

1. Introduction

In real-world topology optimization problems, the optimal performance obtained using conventional deterministic methods can be dramatically degraded in the presence of sources of uncertainty. The source of uncertainty may be the variability of applied loads, spatial positions of nodes, material properties, and so on. Various robust deterministic or probabilistic approaches have been developed to account for different types of uncertainty in structural design and optimization methods (see, for example, Choi et al. [1], Lógó and Pintér [2], and Kharmanda [3]). The interested reader is directed to Bendsøe and Sigmund [4] or Maute [5], which contains an extensive bibliography on this subject. This paper is an extended version of our CSC 2015 paper [6].

Popular choices for robust objective functions are to minimize the expected or maximum performance measure and both approaches have been used when solving the classic compliance minimization problem with uncertain variables (see Califoire et al. [7] and de Gournay et al. [8]). Various parameters can affect the robustness and reliability of a structure, including loading, geometry and material properties. Loading uncertainties are most widely studied. Reliability-based approaches for this uncertainty type are presented by Mogami et al. [9] and Kang and Luo [10]. Methods for approximating probabilistic directional uncertainties are presented by Conti et al. [11], Calafiore et al. [7], Evgrafov et al. [12], and Chen et al. [13].

Dunning et al. [14] proposed a “pseudo-load-oriented” efficient parametric method for considering loading magnitude and directional uncertainty in topology optimization in order to produce robust

* Corresponding author.
E-mail address: csebfalvi@mik.pte.hu (A. Csébfalvi).

https://doi.org/10.1016/j.advengsoft.2018.02.003
Received 14 May 2017; Received in revised form 30 December 2017; Accepted 12 February 2018
0965-9978/

Please cite this article as: Csébfalvi, A., Advances in Engineering Software (2018), https://doi.org/10.1016/j.advengsoft.2018.02.003

solutions. The classical compliance-minimization problem is considered with uncertainties being introduced into the objective function and the loading directional uncertainties are described by continuous normal probability functions.

A similar but "reversed" conception was presented in a private communication paper of Kočvara [15] that defines the optimal robust solution as the design in which the minimum of the maximal compliance on the set of load perturbations satisfies the compliance constraint and the volume fraction is minimal. Borrowing the robustness criterion from Kočvara [15], Csébfalvi [16] presented a worst-load-direction-oriented framework which can be applied to a broad class of engineering optimization problems. A robust volume-constrained set-based (non-probabilistic and non-possibilistic) worst-load-direction-oriented minimax-compliance model with uncertain loading directions, presented by Csébfalvi [17], in which the varying load directions are handled as symmetric angle sets around the nominal load directions. The result of the optimization is a robust volume-constrained design which is insensitive to the loading directional uncertainty on the set of feasible load directions as much as possible.

In this paper, in the comparison phase we used alternatively a robust volume-constrained set-based (non-probabilistic and non-possibilistic) worst-load-direction-oriented minimax-compliance model with uncertain loading directions, developed previously by the first author (Csébfalvi [17]), in which the varying load directions are handled as symmetric angle sets around the nominal load directions. The result of the optimization is a robust volume-constrained design which is insensitive to the directional uncertainty of the loads on the set of feasible load directions as much as possible. Using this set-based minimax-compliance model, 3D benchmark results are presented in Csébfalvi [18]. A new exact algorithm was presented by Csébfalvi [19] for the volume-constrained expected-compliance-minimization problem with normally distributed loading directions using exact objective and gradient functions. The algorithm is based upon the finding that for a particular set of statistical parameters the integration in the expected compliance function can be done symbolically and automatically using symbolic manipulation software.

The parametric expected-compliance statistic is usually labeled as the "preferred measure of robustness" in the topology optimization literature. This may be illustrated by the following citation from the work of Zhao and Wang [20], p. 399: *The main purpose of this work is to consider structural topology optimization under loading uncertainty. To obtain a robust structure, this work will follow the common approach to minimize the expected compliance* [13,14,21–24]. *This has been proven effective to produce a robust structure with good average performance.* In the citation we replaced the original reference numbers with the currently good ones.

According to our opinion the most important element of the citation is the "*a robust structure with good average performance*", which is, in our interpretation, a very subjective evaluation criterion with two very important open questions:

1 What is behind the good average performance?
2 How robust the design is when its performance measure is the average?

Without knowing the exact answers the good average performance may mean nothing from engineering point of view, because even a small directional perturbation can cause large change in the compliance function shape.

In this paper, we will proof indirectly that the expected-compliance-oriented parametric approach is far from the engineering practice and may give hardly interpretable or totally misleading results, which will be demonstrated by two carefully selected counter-examples. The counter-examples validate the fact that the expected-compliance, as a statistical abstraction based on a hypothetical assumption about the directional normality, not a general applicable measure of robustness.

This paper is organized as follows. In Section 2 we describe such models and algorithms which form the methodological basis of the indirect proof. The design examples used as counter examples to demonstrate the deficiencies of the expected-compliance model are presented in Section 3. Finally, some concluding remarks are presented in Section 4.

2. Models and algorithms

In this section, we describe firstly the traditional deterministic volume-constrained compliance-minimization model for single load case and its extension to multi-load cases. After that, we present an exact volume-constrained pure probabilistic expected-compliance-minimizing model where the uncertain loading directions are statistically independent normally distributed variables and a volume-constrained pure nonparametric set-based (sometimes called as interval-based) minimax-compliance model which is only used here as a proofing tool in the critical analysis of the expected-compliance-oriented approach. In the case of each model, we highlight of the most relevant algorithmic aspects and implementation details.

2.1. Deterministic volume-constrained compliance minimization

The mathematical formulation of the traditional deterministic volume-constrained compliance minimization problem is the following:

$$c(\boldsymbol{x}) = \boldsymbol{U}'\boldsymbol{K}\boldsymbol{U} \rightarrow \ \boldsymbol{min} \tag{1}$$

$$V(\boldsymbol{x}) = \varphi \ V_0 \tag{2}$$

$$\boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \tag{3}$$

$$0 \le \boldsymbol{x} \le 1 \tag{4}$$

where $\boldsymbol{x}$ is the vector of design variables (the element densities), $c(\boldsymbol{x})$ is the compliance, $\boldsymbol{U}$ and $\boldsymbol{F}$ are the global displacement and load vectors, respectively, $\boldsymbol{K}$ is the global stiffness matrix, $V(\boldsymbol{x})$ and V_0 are the material volume and design domain volume, respectively, and ϕ is the prescribed volume fraction.

In the case of 2D topology optimization problems, the design domain is assumed to be rectangular and discretized with $n = e^x \times e^y$ square elements discretized with four nodes per element and two degrees of freedoms (DOFs) per node. Both nodes and elements are numbered column-wise from left to right.

As it was demonstrated by Andreassen et al. [26], it is very easy to extend the model for multiple load cases. In the case of m load cases, the force and displacement vectors can be defined as m column vectors and the objective function will be the sum of m compliances:

$$c(\boldsymbol{x}) = \sum_{i=1}^{m} \boldsymbol{U}'_i \boldsymbol{K}\boldsymbol{U}_i \rightarrow \ \boldsymbol{min} \tag{5}$$

$$V(\boldsymbol{x}) = \varphi \ V_0 \tag{6}$$

$$\boldsymbol{K}\boldsymbol{U}_i = \boldsymbol{F}_i, \ i \in \{1, 2, \ldots, m\} \tag{7}$$

$$0 \le \boldsymbol{x} \le 1 \tag{8}$$

The optimization problem (1)–(4) and (5)–(8) can be solved by the well-known optimality criteria (OC) method. See, for example, Sigmund [25] and Andreassen et al. [26]. Naturally, this method can be replaced by any other appropriate solver.