Advances in Engineering Software 119 (2018) 93–102

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Research paper

Fast sensitivity reanalysis methods assisted by Independent Coefficients and Indirect Factorization Updating strategies

Liu Juanjuan[a,b], Wang Hu[a,b,*]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, China
[b] Joint Center for Intelligent New Energy Vehicle, Shanghai, China

ARTICLE INFO

Keyword:
Sensitivity reanalysis
Independent Coefficients
Indirect Factorization Updating
Combined Approximation

ABSTRACT

In this study, the novel sensitivity reanalysis methods are suggested to obtain the static displacement sensitivity to local modifications efficiently. Details of displacement sensitivity formulations are deduced from the displacement reanalysis equations of Independent Coefficients (IC) and Indirect Factorization Updating (IFU) strategies. Moreover, the performances of the proposed sensitivity reanalysis methods are investigated to compare with Combined Approximation (CA) methods. Different numerical examples and efficient comparison are performed to confirm the accuracy and efficiency of the proposed methods. As a result, the proposed sensitivity reanalysis methods are proved to be more efficient with large scale problems and the accuracy of modified Degrees of Freedoms (DOFs) can be guaranteed as well.

1. Introduction

The Sensitivity Analysis (SA) is to evaluate the derivative of design variable. The application of the SA has been extended to the model updating, and damages monitoring or identification and design optimization, etc. [1–5]. In the structural optimization, the SA is used to provide objective and constraint functions of the first or the second derivative information. Generally, the SA is an efficient procedure to improve the accuracy, efficiency and applicability of the algorithm [6], and it can also be used to:

1. Provide the gradient information for approximation technology;
2. Assess the effect of structure properties on the structure response;
3. Provide the search direction in design optimization problems;
4. Generate the approximations response of optimization structure.

The analytic method and finite-difference method (FDM) are fundamental SA methods for the static problem [7]. The analytic method includes direct method (DM) and adjoint-variable method. The DM is costly due to complex and substantial derivation processes [8,9]. Therefore, the semi-analytical method has been proposed to overcome these deficiencies [10,11]. Compared to the DM, the semi-analytical method is more efficient but less accurate. Subsequently, several approaches have been made to eliminate the inaccuracies of the semi-analytical method and to extend the semi-analytical method to the multidiscipline. Moreover, the refined semi-analysis method is suggested and applied to linear, linearized buckling, nonlinear, limit point analyses [12,13]. Wang et al. proposed an improved semi-analytical method based on secant stiffness matrix for geometric nonlinear problem [14]. Deriving from the DM, the other analytic methods, such as adjoint-variable method [15], are capable of avoiding the derivation to the implicit variable. Additionally, the improved adjoint-variable method has been applied to nonlinear structures efficiently [16]. The implementation of FDM is much easier than another fundamental SA method [17]. However, the large truncation errors and the disturbance variables cannot be well controlled. Therefore, the derived methods such as complex step method [18,19] have been proposed to avoid the inherent errors in FDM. The complex variable sensitivity method [20], which solves the step size issue in the complex step method of higher order derivatives, was suggested afterward. Additionally, there are also several other derived SA methods. For instance, the SA method beyond the standard computation of the response derivatives, was proposed based on singular value decomposition [21,22]. The pseudo-analytical sensitivity analysis method [23], which is almost analytical, was provided for the shape optimization of continuum structures.

In the SA procedure, repeated solutions from full analysis usually require much computational effort. To alleviate this difficulty, a number of approaches of reanalysis have been suggested. One of the most popular approximate reanalysis methods is CA method [24–26]. Kirsch applied the CA method to linear static problems, geometrical change and engineering problem [27–30]. Moreover, the CA method was also extended to the SA for static [24,31], dynamic [32] and

* Corresponding author.
E-mail addresses: jjliu@hnu.edu.cn (J. Liu), wanghu@hnu.edu.cn (H. Wang).

https://doi.org/10.1016/j.advengsoft.2018.02.009
Received 21 June 2017; Received in revised form 5 December 2017; Accepted 25 February 2018
0965-9978/

vibration problems [31]. Unlike common approximations of the structural sensitivity, Kirsch and Papalambros [33] used the results of the initial design to solve the modified displacement and structure sensitivity, which firstly verified the uniformity of the CA reanalysis in displacements and displacement derivatives. However, the cost of computational time is large. To reduce the repeated computation in the SA, Kirsch et al. applied the combination of the CA method and differential method to sensitivity reanalysis [34,35]. However, only the sensitivity of the initial structure is solved in their method. Based on the uniformity of CA method in displacement and displacement sensitivity, Zuo et al. proposed an efficient CA sensitivity reanalysis to calculate the modified structure sensitivity for both static and vibration problems [36,37]. Zuo's method may largely increase efficiency with slight loss of accuracy of the sensitivity result.

However, the repeatedly matrix decomposition information which increases memory consumption significantly is required in the CA, especially for large-scale mechanical systems. Other reanalysis techniques, such as block-based reanalysis method [38] and substructuring approaches [39–41], have been successfully applied to the analyses of regular large-scale mechanical systems with locally or globally. Subsequently, the IC method [42] was proposed by Huang et al. for local modification of large-scale structure, which only requires the initial solution as an input. Moreover, it has been successfully applied to the practical complicated applications [43], while the IC method is not a universal numerical method, for instance, the case of local modification of material parameters inside the structure, the corresponding results cannot be reflected totally by the modified stiffness matrix [44]. Based on the IC method and the Sherman-Morrison-Woodbury (SMW) formula, Huang et al. also developed an exact reanalysis method named IFU method, which can efficiently solve the boundary modifications [44]. It is well known that the SMW formula [45,46] is the most basic DM which can obtain the theoretical solution, and it has been greatly extended to multiple rank-one and multiple-rank modifications [47,48]. Thus, there is no theoretical error between IFU reanalysis and complete analysis. However, the IC and IFU methods are not applied to the SA yet. Therefore, the SA formulations of the IC and IFU are deduced based on the uniformity of reanalysis procedures for evaluating displacements and first-derivatives of displacement in this study. Moreover, we hope to compare the performances of the CA, IC and IFU and investigate the characteristics of these reanalysis-based SA methods.

The rest of this paper is organized as follows. The CA sensitivity method proposed by Zuo et al. is briefly reviewed firstly in Section 2. The IC and IFU are briefly introduced and corresponding sensitivity reanalysis formations are deduced in Section 3 and Section 4, respectively. The efficiency is demonstrated by the Number of Algebraic Operations (NAO) of three sensitivity reanalysis methods in Section 5. Three numerical examples are employed to demonstrate the accuracy of three sensitivity methods in Section 6. The conclusion is summarized in the final section.

2. CA sensitivity reanalysis

The CA sensitivity method proposed by Zuo et al. [37] has been successfully verified for static and vibration problems, which makes up the deficiency of the Kirsch's method in the frame of guaranteeing the accuracy and efficiency. The procedure of CA sensitivity method for static is briefly reviewed to facilitate performance comparison with the proposed sensitivity reanalysis procedures.

Given an initial design, including the initial stiffness matrix $\boldsymbol{K}_0$ and the load vector, the static displacement $\boldsymbol{r}_0$ can be computed by the equation

$$\boldsymbol{K}_0\boldsymbol{r}_0 = \boldsymbol{F} \tag{1}$$

where the chosen design variable is unrelated to the load $\boldsymbol{F}$. The initial displacement sensitivity $\partial\boldsymbol{r}_0/\partial x_i$ with respect to the ith design variable x can be calculated

$$\boldsymbol{K}_0\frac{\partial\boldsymbol{r}_0}{\partial {x_i}^0} = -\frac{\partial\boldsymbol{K}_0}{\partial {x_i}^0}\boldsymbol{r}_0. \tag{2}$$

Assume a change in the initial design, the modified displacement sensitivity $\partial\boldsymbol{r}/\partial x_i$ can be calculated by the equilibrium equations presented as

$$(\boldsymbol{K}_0 + \Delta\boldsymbol{K})\frac{\partial\boldsymbol{r}}{\partial x_i} = -\frac{\partial\boldsymbol{K}}{\partial x_i}\tilde{\boldsymbol{r}}, \tag{3}$$

$$\boldsymbol{r} \approx \tilde{\boldsymbol{r}} = \boldsymbol{r}_0 + \frac{\partial\boldsymbol{r}_0}{\partial x}(\boldsymbol{x} - \boldsymbol{x}_0)^T. \tag{4}$$

The CA sensitivity method proposed by Zuo et al. uses the calculation of the first-order Taylor series expansion $\tilde{\boldsymbol{r}}$ as the modified displacement $\boldsymbol{r}$ in Eq. (4).

Use Eq. (3) to solve the modified response sensitivity $\partial\boldsymbol{r}/\partial x_i$, and the required steps are outlined as follows:

(1) Construct the basis vectors of displacement sensitivity by using the first s items from Neumann series expansion. The final iteration form without derivation is directly given as

$$\frac{\partial\boldsymbol{r}}{\partial x_i} = (\boldsymbol{E} + \boldsymbol{D})^{-1}\boldsymbol{r}_1 = (\boldsymbol{E} - \boldsymbol{D} + \boldsymbol{D}^2 - \boldsymbol{D}^3 + \ldots)\boldsymbol{r}_1, \tag{5}$$

$$\boldsymbol{r}_B = (\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_s) = (\boldsymbol{r}_1, -\boldsymbol{D}\boldsymbol{r}_1, \ldots, -(-1)^s\boldsymbol{D}^s\boldsymbol{r}_1), \tag{6}$$

The symbol of $\boldsymbol{E}$ represents the unit matrix and $\boldsymbol{r}_1 = -{\boldsymbol{K}_0}^{-1}\frac{\partial\boldsymbol{K}}{\partial x_i}\tilde{\boldsymbol{r}}$, $\boldsymbol{D} = {\boldsymbol{K}_0}^{-1}\Delta\boldsymbol{K}$ in Eqs. (5) and (6).

(2) Construct the reduced equation of the displacement sensitivity. In the CA method, the modified displacement sensitivity can be approximated by the linear combination of basis vectors as

$$\frac{\partial\boldsymbol{r}}{\partial x_i} \approx \frac{\partial\tilde{\boldsymbol{r}}}{\partial x_i} = y_1\boldsymbol{r}_1 + y_2\boldsymbol{r}_2 + \ldots + y_s\boldsymbol{r}_s = \boldsymbol{r}_B\boldsymbol{y}. \tag{7}$$

Substitute Eq. (7) into Eq. (3) and pre-multiply ${\boldsymbol{r}_B}^T$ on both sides of the equation to obtain

$${\boldsymbol{r}_B}^T\boldsymbol{K}\boldsymbol{r}_B\boldsymbol{y} = -{\boldsymbol{r}_B}^T\frac{\partial\boldsymbol{K}}{\partial x_i}\tilde{\boldsymbol{r}}. \tag{8}$$

Calculate the reduced equation which is consist of s dimension linear equations. After solving for the coefficient $\boldsymbol{y}$, substitute it into Eq. (7), the modified displacement sensitivity $\partial\boldsymbol{r}/\partial x_i$ can be obtained.

In order to evaluate the accuracy of sensitivity reanalysis method, the normalized error can be obtained as

$$\boldsymbol{\varepsilon} = \left(\frac{\partial\tilde{\boldsymbol{r}}}{\partial x_i} - \frac{\partial\boldsymbol{r}}{\partial x_i}\right). / \frac{\partial\boldsymbol{r}}{\partial x_i}, \tag{9}$$

where $\partial\tilde{\boldsymbol{r}}/\partial x_i$ is the displacement sensitivity solved by the CA method, and $\partial\boldsymbol{r}/\partial x_i$ is the displacement sensitivity without reanalysis.

3. Independent coefficient sensitivity reanalysis

In this section, we proposed a novel sensitivity reanalysis in the frame of the IC strategy. Therefore, the procedures of IC method [42] are introduced briefly, then the corresponding sensitivity reanalysis formulas are deduced.

3.1. IC reanalysis

Assuming that the equilibrium equations of initial design are

$$\boldsymbol{K}_0\boldsymbol{r}_0 = \boldsymbol{F}. \tag{10}$$

where initial displacements can be obtained by whatever available