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Research paper

An adaptive isogeometric boundary element method for predicting the effective thermal conductivity of steady state heterogeneity

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ABSTRACT

This work presents an adaptive isogeometric boundary element method (IGBEM) for the calculation of effective thermal conductivity of steady state heterogeneities. Based on the generalized self-consistent scheme (GSCS), some integral equation formulations which only contain the unknown temperatures on the interface are used to calculate the effective thermal conductivity of steady state composites. In our approach, the geometries of the inclusion and original matrix are described using NURBS basis functions. The advantage over currently used methods is that no geometrical errors exist in the analysis process. And the geometry data in the isogeometric GSCS model can be taken directly from CAD programs. In addition, based on the upper bound of the relative error of the Gaussian quadrature formula, an adaptive integration method is used to compute the boundary integrals, which makes the computation of the integrals easier and more efficient at optimal computational cost. The comparisons between the results obtained by the present method and the existing counterparts are carried out and the good agreement can be observed.

1. Introduction

Composite materials are being used increasingly in a variety of modern engineering applications and this trend is likely to continue. The reason is that many composite materials possess a number of highly desirable engineering properties that can be exploited to design structures with high demand on their performance. Therefore, analysis of the effective properties of composite materials has received considerable attention in scientific community [1]. Many theoretical models, such as differential scheme [2], modified Eshelby's model (MEM) [3], self-consistent method (SCM) [4] and the generalized self-consistent scheme (GSCS) [5], for predicting the effective properties of composites have been presented. In [6,7], the SCM was applied to a composite reinforced with spherical fillers, which determines the elastic constants of the composite by embedding only one filler into an infinite domain with the composite property determined. The GSCS, closely related with the SCM, has been proposed by Christen and Lo [5]. Main idea of the GSCS is in the assumption that the particle surrounded by the matrix material is embedded in an effective medium of unknown properties. This method can yield better results than the SCM [8].

The finite element analysis of the GSCS for solving the elastic-plastic heterogeneous problems, inverse problems and mechanical degradation of fibrous composites was carried out by Lefik et al. [9] and Boso et al. [10,11]. This method is based on two separated finite element models, i.e. a fine heterogeneous model and a coarse homogeneous model. The effective material properties of a coarse homogeneous model are iteratively computed enough to close the response of the fine heterogeneous. The disadvantage of this method is that it requires to discretize the whole computation domain and more fine meshes near the interfaces, i.e. the inclusion and the original matrix as well as the original matrix and the effective matrix, are needed. Therefore, a huge computer memory is needed to store the related finite element information. And lots of computer execution time is required to calculate the effective properties of composited material. Due to the merits of high accuracy and only the boundary description of the problem, the boundary element method (BEM) has been widely used to deal with the thermal conduction problems [12–13]. In [12], each region with different heat transfer properties was taken as a piecewise homogeneous in a heterogeneous medium. The resulting non-square global system matrix was solved by the singular value decomposition method. In [13], some new integral equation formulations suitable for steady state thermal conduction were presented to calculate the effective thermal conductivity of steady state problems. These equations only contain the unknown temperatures on the interface. Boundary face method was used to deal with thermal conduction problems in [14]. And a large number of open-ended tubular shaped holes of small diameters were studied. In numerical process, a new meshing scheme was adopted to discretize the holes of which the exact geometry remained.

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In 2005, Hughes et al. proposed the ``Isogeometric Analysis'' (IGA) paradigm [15] as a means to perform finite element analysis directly from computer aided design data, for three-dimensional regions. The first paper known to the authors to propose isogeometric approximations dates back to 1982 [16], although the approach was significantly different from the 2005 paper of Hughes et al. Several methods were later devised in order to alleviate the difficulties existed in the original version of IGA. Particularly the lack of an automatic parameterisation to build the approximation within the domain was addressed through (a) special parameterisation techniques, for example based on variational harmonic methods [17]. Xu et al. proposed a method to parameterize the computational domains [18]. Xu et al. also devised analysis-aware parameterisation methods for single [19] and multi-domain geometries [20]. The stability issues associated with parameterization was studied in [21]. (b) geometry independent field approximation (GIFT) where the spline spaces used for the geometry and the field variables can be chosen and adapted independently while preserving geometric exactness and tight CAD integration [22]. (c) isogeometric boundary element methods (IGBEM), proposed in [23,24] and was later generalized to 3D T-spline geometries in [25]. IGBEM allows stress analysis directly from CAD, without any mesh generation or regeneration [26] and was recently used for damage tolerance assessment of complex structures, directly from CAD [27,28]. IGBEM was also used for 2D and 3D shape optimizations in [29,30].

A review of IGA was proposed in paper by Nguyen et al. [31] and a review of recent efforts to streamline the CAD-analysis transition pipeline was provided in [32,33]. Note that a wide range of other methods, relying also directly on CAD are also aiming at CAD-analysis integration. One should, in particular, refer to the work of Sevilla et al. [34], Moumnassi et al. [35] and Legrain et al. [36–38]. But significantly complicated adaptive h-refinement since tensor product approximations are still common place. This means that the approximation must be refined everywhere at once in the domain. Alternatives to this were proposed by [22,39], where the geometry and the field approximations are independent. Similar ideas were proposed earlier by Sevilla et al. [34] and allowed to obtain exact boundary representation, as in IGA, but without requiring the interior discretisation to be tied to the geometry representation, offering more flexibility. A second difficulty encountered by IGA is the need for an interior parameterisation to be constructed from the CAD data, which only provides boundary information. Significant work was already performed to achieve this, notably [40] where collocation methods/meshfree approximations are constructed within the domain whilst preserving geometry exactness. Finally, isogeometric boundary element methods [23,24] are probably the best suited candidates to overcome this interior discretisation obstacle, since only boundary data is required for analysis, which enables, for example, stress analysis [23,25], acoustic problems [41], potential problems [42,43] and damage tolerance/crack propagation analysis [27] to be performed without any mesh generation step, directly from CAD.

In this paper, the adaptive integration scheme based on sub-division technique presented in [44] is coupled with the IGBEM to control the numerical error of the integration. Adaptive scheme accounts for nearly singular and singular integrals existing in BEM problems [25,43]. In [45,46], an adaptive scheme was introduced for fracture problems. Cirak et al. proposed in 2000 a method based on subdivision surfaces for thin-shell finite element analysis [47]. In 2002, Cirak et al. [48] also proposed an integrated modeling FEA and engineering design approach for thin-shell structures using subdivision surfaces. This work is concerned with the calculation of the effective thermal conductivity of steady state heterogeneities using an adaptive IGBEM.

The composite considered in this work is assumed to be misoriented in space, namely, statistically homogeneous [49,50]. Following the method in [13], the integral equation formulations which only contain the unknown temperatures on the interface are used in the implementation of the IGBEM. And a heat energy computation formulation which only contains the interface integrals is adopted to calculate the system heat energy. Based on the GSCS model, some numerical examples are solved. The present results are compared with the exact solutions or upper and lower bounds of solutions. The results show the accuracy and effectiveness of the present method.

A short description of the contents of this paper is as follows. Section 2 introduces necessary background concepts about GSCS and the differential formulations of the physical problem. Section 3 presents the formulations for isogeometric GSCS model. Section 4 gives the adaptive integration scheme for boundary integrals on isogeometric element. In Section 5, the computation process of the isogeometric GSCS is described. Several numerical examples are given in Section 6 to verify the efficiency and accuracy of the present method. Finally, we present the conclusions for our work.

2. Problem statement

GSCS takes into account the interaction between matrix and inclusions by considering a representative unit cell inclusion, i.e., an inclusion and a surrounding matrix, which is itself embedded in the infinite effective matrix. Owing to considering the full range of the volume fraction of inclusion, it gives a physically realistic model of inclusion to inclusion interaction for two-phase system. The generalized self-consistent model of the presented problem is shown in Fig. 1, where the geometry of the model is described by NURBS. More details about principles of GSCS model can be found in [5,9–11,13].

As shown in Fig. 1, assume that Γ_1 and Γ_2 represent the inclusion-original matrix interface and the original matrix-effective matrix interface, respectively. The heat fluxes along *x, y* and *z* axes are indicated by q_x,q_y and q_z, respectively. Here, we focus our attention on the numerical implementation of the adaptive IGBEM for calculation of effective heat conductivity of steady state heterogeneity. Thus, only some conclusions are given, more details about basic idea and derivation process of boundary integral equation can be found in [13].

According to several boundary integral equations obtained by the location of the source points and continuity condition [13], the following formulas which only contain the unknown temperatures on the interface can be obtained. When the source point *P* is on Γ_2, we have the following integral equation:

$$\left(c_E(P) + \frac{k_M}{k_E}c_M(P)\right)u_E(P) = u^0(P) - \int_{\Gamma_2}\left(1 - \frac{k_M}{k_E}\right)T_E(P, q)u_E(q)d\Gamma - \int_{\Gamma_1}\left(\frac{k_M - k_I}{k_E}\right)T_M(P, q)u_M(q)d\Gamma \tag{1}$$

where k_I, k_M and k_E are the thermal conduction coefficients of the

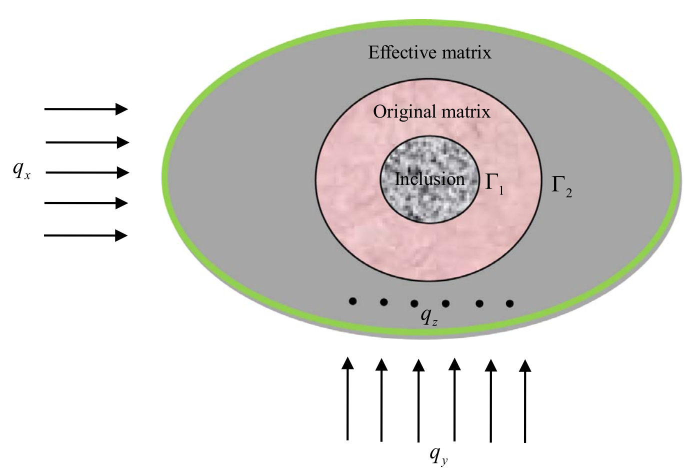

Fig. 1. The GSCS model with single inclusion.