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Development of a finite element/discontinuous Galerkin/level set approach for the simulation of incompressible two phase flow

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ABSTRACT

In this paper, we present a combined finite element/ discontinuous Galerkin/ level set method to simulate the incompressible two phase flow. The level set method is employed to capture the moving interface because of its simplicity and efficiency when dealing with the significant interface deformations. Due to the hyperbolic nature of the level set equation and the level set re-initialization equation, we apply the second order Runge Kutta Discontinuous Galerkin (RKDG) method to get the stable and precise results. Moreover, in order to obtain the accurate velocity field, the hybrid continuous and discontinuous Galerkin approach is utilized to solve the incompressible Navier–Stokes equations. In our combined method, there is no need to re-initialize the level set function in every time step and the re-initialization process is implemented after suitable time steps. The desirable mass conservation property is able to be guaranteed with a mass correction technique in the combined algorithm. In addition, the stabilization terms can be avoided in the whole computational process. Several challenging problems, i.e., a rising bubble, the dam-break flow, the Rayleigh–Taylor instability, and the complex metal casting process in the engineering applications are investigated to evaluate the feasibility, validity and practicability of our approach for solving the problem of the incompressible two phase flow.

1. Introduction

Two phase flow appears in numerous engineering applications, such as metal casting, wave mechanics, water-oil separation, and the crystal growth. In the finite element framework, developing the effective numerical approach to simulate the flow with free surface via the level set method [1,2] is of great importance and has been a research hotspot in the past several decades [3–8].

In the simulation of the incompressible two phase flow, the surface tension plays an important role in some cases, and the interface experiences significant deformations due to the large density and viscosity ratios. The level set method is easy to deal with the conspicuous interface deformations. It is also convenient to calculate the outer normal direction of the interface via the level set function, which is essential for computing the surface tension. Therefore, the level set approach is commonly used to capture the moving interface. However, solving the level set and the level set re-initialization equations utilizing the standard Finite Element Method (FEM) may lead to the numerical oscillations [9]. In 2005, Nagrath et al. [10] proposed a streamline-upwind/ Petrov–Galerkin (SUPG) method to solve the level set equation. Whereas, the level set function needs to be re-distanced at the end of each time step [10,11], which consumes more computational time. In 2016, Touré et al. [12] transformed the level set equation into a convection-diffusion form with parameters and resorted to the stabilized SUPG method for the spatial discretization. Nevertheless, it is not easy to deal with the boundary conditions. The computation of the level set equation in the switched form also needs extra stabilization term and it may cause some unexpected numerical errors.

The Discontinuous Galerkin (DG) method [13–15], which was originally introduced for the hyperbolic type equations, is an attractive approach to deal with the level set and the level set re-initialization equations [16] for solving the problem of the incompressible two phase flow. However, when utilizing DG to handle the elliptic equation [17], it increases the memory size, the complexity for programming and the total computational cost.

In addition, the level set method has the main drawback of the mass loss or gain [18]. Thus, the authors [19] established the combination of the volume of fluid method and the level set method to enforce the mass conservation property. However, the simplicity of the original level set algorithm lost in this case.

As for the engineering problem with incompressible two phase flow, it usually involves the irregular computational domains. In the simulation of the metal casting process, Pang et al. [20] have simplified the geometry of the irregular cavity to reduce the difficulty of dealing with

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the boundary conditions. However, it lowers the authenticity of the final results.

In this work, our main goal is to advocate a combined finite element/discontinuous Galerkin/level set approach to tackle the incompressible two phase flows efficiently, stably and accurately. This combined approach is also applied to model the complex metal casting for the irregular cavity in the engineering. In the combined algorithm, we make the best of the advantages of FEM and DG to avoid the stabilization terms and save the total computational cost. In this way, we employ the continuous and discontinuous Galerkin method based on the split scheme [21] to solve the incompressible two phase Navier-–Stokes equations and obtain the precise velocity field. Furthermore, the second order RKDG method is used to deal with the level set and the level set re-initialization equations to achieve the accurate and stable results. As a result, the level set re-initialization procedure is not necessary in every time step. We are able to carry out the level set re-initialization equation after a certain number of time steps and the computational cost is reduced. In addition, we add a simple geometric mass correction algorithm [11] in our combined method to overcome the major shortcoming of the gaining or losing mass during the computation of the level set method.

The paper is organized as follows. In Section 2, we briefly introduce the non-dimensional incompressible Navier–Stokes equations, the level set equation and the level set re-initialization equation. In Section 3, we describe our combined method in details. Then, in Section 4, several challenging problems, namely, a bubble rising, the dam-break flow, the Rayleigh–Taylor instability and the complex metal casting process are investigated to demonstrate the feasibility, validity, robustness and practicability of our combined method. Finally, in Section 5, we give some concluding remarks.

2. Governing equations

The incompressible two phase flow problem always contains two different fluids and the moving interface. A schematic graph is shown in Fig. 1. The area of fluid 1 and fluid 2 are denoted as Ω_1 and Ω_2, respectively. The interface between these two fluids is represented by Γ. Let us assume that both fluids are viscous and Newtonian. The mathematical model of this problem mainly involves the incompressible Navier–Stokes equations and the level set equation.

2.1. Non-dimensional incompressible Navier–Stokes equations

The non-dimensional incompressible Navier–Stokes equations of the two phase flow can be written as [22,23]

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla\mathbf{u}) = -\frac{\nabla p}{\rho} + \frac{1}{\rho Re}\nabla\cdot(\mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}})) + \frac{1}{Fr^2}\mathbf{g}_0 + \frac{1}{\rho Wb}\mathbf{F}_{sv} \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{2}$$

where $Re = \frac{\rho_{ref} L_{ref} U_{ref}}{\mu_{ref}}$, $Fr = \frac{U_{ref}}{\sqrt{g L_{ref}}}$, $Wb = \frac{\rho_{ref} U_{ref}^2 L_{ref}}{\sigma}$, Re represents the Reynolds number, Fr denotes the Froude number, Wb is the Weber number, g means the gravitational acceleration, σ is the coefficient of surface tension, $\mathbf{g_0}$ is the unit vector in the direction of gravitation, and U_{ref}, L_{ref}, ρ_{ref}, μ_{ref} are the reference velocity, length, density and viscosity, respectively. The velocity is denoted by $\mathbf{u}$, p is the pressure, ρ is the density and μ is the viscosity. The force due to the surface tension on the interface is given by Brackbill et al. [24]

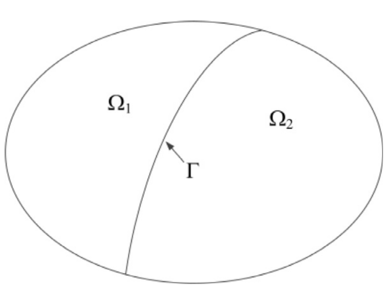

Fig. 1. A sketch of the phase flow problem.

$$\mathbf{F}_{sv} = -\left(\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|}\right)\nabla\phi \tag{3}$$

where ϕ is the level set function.

The density and viscosity are able to be written on the entire domain [22,23] as

$$\rho(\phi) = \rho_1 + (\rho_2 - \rho_1)H(\phi) \tag{4}$$

$$\mu(\phi) = \mu_1 + (\mu_2 - \mu_1)H(\phi) \tag{5}$$

where ρ_1 and ρ_2 are the density of fluid 1 and fluid 2; μ_1 and μ_2 are the viscosity of fluid 1 and fluid 2. The smoothed Heaviside function, $H(\phi)$, is written as follows

$$H(\phi) = \begin{cases} 0, & \phi < -\varepsilon \\ \frac{1}{2}\left(1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{\varepsilon}\right)\right) & -\varepsilon \le \phi \le \varepsilon. \\ 1, & \phi > \varepsilon \end{cases} \tag{6}$$

and we choose ε to be about $1.3h_{\max}$ in the calculation, where $h_{\max}$ is the maximum grid size.

2.2. Level set function

In our algorithm, a level set function ϕ is utilized to capture the interface. The values of ϕ are set as negative in the fluid area of Ω_1 and positive in the fluid area of Ω_2. The interface Γ is represented by the zero level set of ϕ

$$\Gamma = \{\mathbf{x}|\phi(\mathbf{x}, t) = 0\} \tag{7}$$

With time evolving, the level set function is advected by the fluid velocity field. As for the incompressible two phase flow, the level set equation in the conservative form is that

$$\frac{\partial\phi}{\partial t} + \nabla\cdot(\mathbf{u}\phi) = 0 \tag{8}$$

2.3. Re-initialization equation of level set function

In order to make sure that the level set function remains to be the signed distance function, it is necessary to solve the following equation for a few steps with the initial condition [1]

$$\begin{cases} \frac{\partial\phi}{\partial\tau} = sign(\phi_0)(1 - |\nabla\phi|) \\ \phi(\mathbf{x}, 0) = \phi_0(\mathbf{x}) \end{cases} \tag{9}$$

where $sign(\phi_0) = \frac{\phi_0}{\sqrt{{\phi_0}^2 + \varepsilon^2}}$, ϕ_0 is the initial level set to be re-initialized, τ represents the artificial time and ε is half of the interfacial thickness. The artificial time step Δτ is chosen as the same with Δt. The boundary condition is that $\mathbf{n}_{\text{wall}}\cdot\nabla\varphi = 0$, where $\mathbf{n}_{\text{wall}}$ is the outward normal direction of the wall.

We can rewrite the equation as

$$\frac{\partial\phi}{\partial\tau} + \mathbf{w}\cdot\nabla\phi = sign(\phi_0) \tag{10}$$

where $\mathbf{w} = sign(\phi_0)\frac{\nabla\phi}{|\nabla\phi|}$.