Advances in Engineering Software 000 (2017) 1–6

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

An iterative method for the linearization of nonlinear failure criteria

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ARTICLE INFO

Article history:
Received 16 February 2017
Accepted 24 April 2017
Available online xxx

Keywords:
Elastoplastic analysis
Finite element method
Nonlinear failure criteria
Linearization
Stress analysis
Hoek–Brown failure criterion

ABSTRACT

Linearization of a nonlinear failure criterion may be required if a software incorporating the nonlinear failure criterion is not readily available or if an additional computational difficulty is encountered due to the nonlinearity of the failure criterion. In this paper, a novel method is proposed for the linearization of nonlinear failure criteria. The proposed method is based on an iterative procedure and the least-squares regression is used for the linearization. The material is assumed to be elastic-perfectly plastic. By conducting analytical and finite element analyses of stresses and displacements around underground openings in rock mass governed by the generalized Hoek–Brown failure criterion, it is shown that the proposed method is convergent, efficient and effective.

1. Introduction

For the elastoplastic finite element analysis of solids encountered in structural or geotechnical engineering, the material properties may be defined more accurately by using a nonlinear failure criterion [1–3] as shown in Fig. 1. In this figure, σ_1 and σ_3 are, respectively, the major and minor principal stresses and f is a nonlinear function of σ_3. The problem of non-convergence of non-elastic problems in implicit analysis is quite common. The nonlinearity of the failure criterion and the yield function may lead to additional problem of non-convergence. Therefore, an otherwise converging solution may become non-convergent due to the nonlinearity of the failure criterion [4–6]. Furthermore, due to this and other reasons, a suitable software incorporating the nonlinear failure criterion may not be readily available.

For example, the conventional linear Mohr–Coulomb (M-C) failure criterion is the most commonly used failure criterion in geotechnical engineering and is implemented in most of the general purpose finite element software. However, this failure criterion is not suitable for many types of rocks, particularly a jointed rock mass. For such rocks, the nonlinear generalized Hoek–Brown (GH-B) failure criterion [2] is more appropriate. To the author's knowledge, this failure criterion has not been implemented in any general purpose finite element software. Some special purpose finite element software, for example, Phase[2] [6] have implemented this failure criterion. However, numerical problems such as non-convergence of results may be encountered if the failure criterion is changed from the linear M-C to the nonlinear GH-B [4–6].

In order to circumvent these computational difficulties, researchers [2,4–5,7–17] have proposed various methods of linearizing the nonlinear failure criterion as shown in Fig. 2 in which g is a linear function of σ_3. Most of these methods are based on the curve-fitting approach within a predetermined range of stress. In the earlier methods [7–8], the linearization was assumed to be independent of the actual state of stress. Later, it was realized that the linearization should be based on the range of stresses existing within the yielded zone and thus improved methods of linearization were proposed [9–17] and applied to different types of problems.

In all such methods, the range of major or minor principal stress within the yielded zone is either assumed or obtained by using available analytical solutions for simple cases. Thus the methods are valid for such specific cases only and for other problems, the results are highly approximate.

Most recently, a general method was proposed [18] for the computation of the equivalent parameters for the linear M-C failure criterion for rock mass governed by the nonlinear GH-B failure criterion.

The objective of this paper is to present the earlier work [18] in a general form so that it could be applied to a wide variety of other problems encountered in structural and geotechnical engineering. The paper has been revised further in the light of the comments and discussions following the presentation of the paper at the conference [18].

In the proposed method, the material behaviour is assumed to be elastic-perfectly plastic (Fig. 3) and the effect of strain-hardening or softening behaviour is not considered. In this figure, ε_1 is the major principal strain. The range of minor or major principal stresses within the yielded zone is determined by using an

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http://dx.doi.org/10.1016/j.advengsoft.2017.04.008

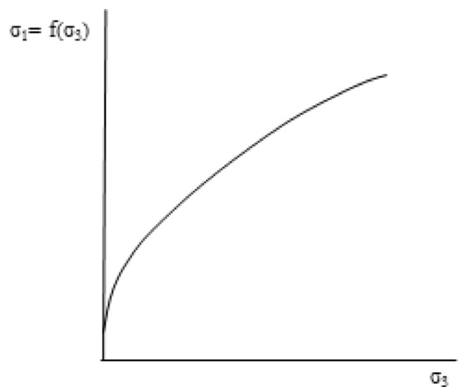

Fig. 1. A nonlinear failure criterion.

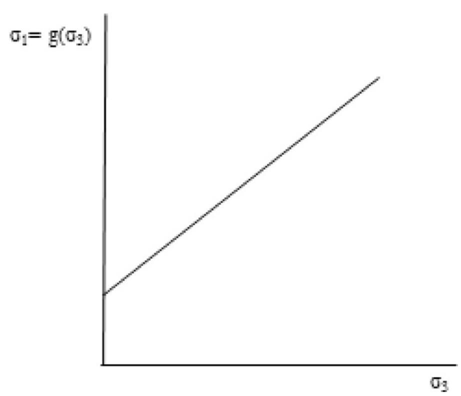

Fig. 2. A linear failure criterion.

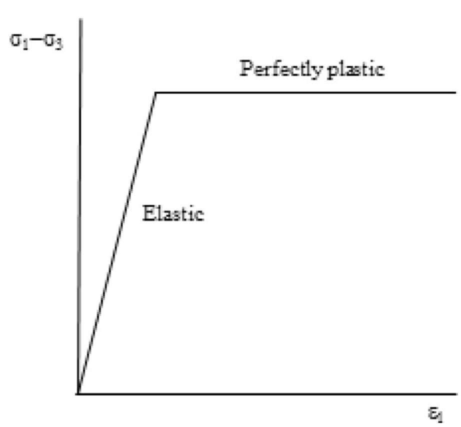

Fig. 3. Elastoplastic model used for the analysis.

iterative procedure. The initial values of the range of minor principal stresses are computed by using an elastic analysis or it may be based on a reasonable estimate as proposed in [17]. The least-squares regression is used for the linearization of the nonlinear yield function within the range computed by using the analytical solution or by using the finite element method. The linear yield function thus obtained is then used to conduct the elastoplastic analysis and the new range of values of the minor or major principal stresses is determined. The iterative process is continued until a convergence criterion is satisfied.

Results of several numerical tests are presented to demonstrate the effectiveness of the proposed method.

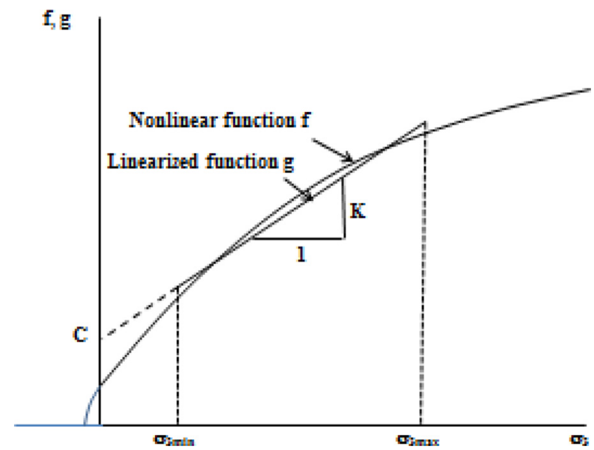

Fig. 4. Linearization of a nonlinear failure criterion.

2. Yield and plastic potential functions

It is assumed that the nonlinear failure criterion is independent of the intermediate principal stress and as mentioned earlier, it may be expressed in the following form:

$$\sigma_1 = f(\sigma_3) \tag{1}$$

The nonlinear function f depends on the mechanical properties of the material. Compressive stresses are considered to be positive. The yield function F is thus expressed as

$$F = \sigma_1 - f(\sigma_3) = 0 \tag{1a}$$

The plastic potential function Q is assumed to be linear and non-associated and is given by

$$Q = \sigma_1 - K_d \sigma_3 \tag{2}$$

where K_d is the dilation parameter of the material. The value of $K_d = 1$ corresponds to a nondilatant material.

3. The proposed method

The nonlinear function f has to be linearized within a particular range of values of σ_1 or σ_3. As mentioned earlier, in the existing methods, the range is either assumed or determined analytically for simplified cases. For higher accuracy, the range of values of σ_1 or σ_3 should be the actual values existing within the yielded zone. However, in general, this is initially unknown. As mentioned earlier, the proposed method is based on an iterative procedure to determine the actual range. Initial values of the range are based on the elastic analysis. Alternatively, as mentioned earlier, the initial values may be based on approximations proposed in [17]. The linearization based on the range of σ_1 requires the solution of a nonlinear equation. Therefore, in the proposed method, it is preferred to use the range of σ_3. For any given range of σ_3 from $\sigma_{3\min}$ to $\sigma_{3\max}$, the function f may be approximated by a linear function g as shown in Fig. 4 and the linearized failure criterion may be expressed as

$$\sigma_1 = g(\sigma_3) = K\sigma_3 + C \tag{3}$$

where K and C are linearization parameters for the material. The corresponding equivalent or virtual yield function is thus given by

$$F_{eq} = \sigma_1 - g(\sigma_3) = 0 \tag{3a}$$

As mentioned earlier, the proposed linearization is based on the least-squares regression and thus K and C are obtained by solving the following system of two linear equations: