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Research paper

A half-analytical elastic solution for 2D analysis of cracked pavements

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ABSTRACT

This paper presents a half-analytical elastic solution convenient for parametric studies of 2D cracked pavements. The pavement structure is reduced to three elastic and homogeneous equivalent layers resting on a soil. In a similar way than the Pasternak's modelling for concrete pavements, the soil is modelled by one layer, named shear layer, connected to Winkler's springs in order to ensure the transfer of shear stresses between the pavement structure and the springs. The whole four-layer system is modelled using a specific model developed for the analysis of delamination in composite materials. It reduces the problem by one dimension and gives access to regular interface stresses between layers at the edge of vertical cracks allowing the initial debonding analysis. In 2D plane strain conditions, a system of twelve-second order differential equations is written analytically. This system is solved numerically by the finite difference method (Newmark) computed in the free Scilab software. The calculus tool allows analysis of the impact of material characteristics changing, loads and locations of cracks in pavements on the distribution of mechanical fields. The approach with fracture mechanic concepts is well suited for practical use and for some subsequent numerical developments in 3D.

1. Introduction

Most of pavement structures subjected to the climatic hazards and repeated passage of heavy loads (bus, truck, aircraft, etc.), reach their limit of life and need rehabilitation. They can be reinforced by several methods such as adding or replacing layers. Depending on the type of roads, these layers are made with different types of materials such as recycling bituminous material, cement concrete layers especially in France for urban area and/or glass grids. Taking into account the possibility of presence of discontinuities in a pavement structure is then important for analysing its final service life and predicting its failure mode [1]. Nowadays, these reinforcements are poorly mastered because they still are generally proposed empirically or on the basis of design methods that do not take into account partial discontinuities at the interfaces between layers and/or vertical cracks in multilayer pavement structures. Indeed, most of the design pavement methods, such as the French method [2], use the elastic Burmister's axisymmetric model [3]. This modelling as well as advance ones developed for 3D viscoelastic pavement tools, for instance ViscoRoute© [4–6], cannot take into account vertical discontinuities and/or partial delamination encountered in these multilayer damaged structures. So far, there is no tool for engineers to calculate and analyse the mechanical fields responsible of the debonding between layers in the cracked pavement structures. The main objective is to propose practical tools for the design of appropriate and sustainable solutions of pavement reinforcement. To be widely used in engineering offices, these tools must be fast and easy to implement in software.

The analysis of multilayer structures partially cracked may be difficult due to singularities located at the interfaces between layers of material near the edges or vertical cracks [7,8]. Many works, preliminary on theoretical modelling [9–11], then on experimental developments [12,13] and on numerical modelling [14] may help to understand how taking into account those discontinuities in such 3D multilayer structures. Among the existing numerical methods, the Finite Element Method (FEM) is nowadays the most used one. This method has many advantages such as introducing complex boundary conditions. But it requires the use of fine meshes near cracks or discontinuities. It increases the work of 3D meshing and the time of calculation. At the end of 90s, advanced numerical models such as G-FEM [15] and X-FEM [16] have been developed in order to overcome the problems of re-meshing during crack propagation. More recently, new models such as TLS (Thick Level Set) approach, developed by [17], permit the description of initiation and propagation of defects in a unified framework [18]. However, the introduction of partial inter-facial cracks is not fully established yet, even if there is many promising works on this subject [19,20]. In addition, all these models and approaches to solve the problem may be too heavy to be contained in practical software.

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In an alternative way, the French Institute of Science and Technology of Transport, Development and Networks (IFSTTAR) proposes an approach that is used in this paper. It uses one of the multi-particle models of multilayer materials (M4) that have been specially developed to study the edge effects and delamination in composite structures materials [21–23]. In such models, assumptions for the unknown field variables are introduced for each layer separately.

The M4 selected herein for the pavement bending problem contains five kinematic fields (5n) per layer i (i ∈ {1, n}, where n denotes the total number of layers). The M4-5n is formulated by making approximations on the stress field. It has a linear polynomial expansion through the thickness, in each layer, for the bending stresses. The Hellinger–Reissner variational principle [24] is used. Then from a multi-particulate description in 2D, this modelling approach can determine the intensity of the 3D mechanical fields. As opposed to other classical models, these mechanical models yield finite stresses at a free edge or crack tip at the interface point location of two different layers. While the number of unknowns may be significant, the semi-analytical solution of the equations of the model allows for easy and quick parametric studies. It is useful for modelling office engineer calculation types [25,26].

For pavement-cracked applications, an initial adaptation of the multi-particular model (M4-5n) combined with the Boussinesq model (B) [27] for the soil has been proposed in [28,29]. The M4-5nB proposed has shown its effectiveness in modelling 3D mechanical fields in the case of loading with or without introduction of thermal gradients. However, the numerical solution of the system of differential equations of order 2 of the M4-5n equations coupled to the integral Boussinesq's equations still takes too long for the expected final 3D tool. A faster solution using Winkler springs [30] for the soil, named M4-5nW, was tested recently, and applied to the case of plane strain of bilayer and tri-layer pavement [31]. The results showed that the stress fields at the interfaces are similar to those obtained by the simulation of a structure with a Boussinesq soil and the FEM solution far from the interface between the pavement and the soil. In that case the M4-5nW solution is obtained four times CPU faster than the M4-5nB and five times faster than the one obtained by the FEM.

Following these previous works [29] and in order to simplify to a maximum the modelling of the real 3D pavement, this one is finally chosen equivalent to 3 layers (surface course, base course and sub-base course) [32] resting on a soil. In the aim to get better approximations of the mechanical fields between the pavement layers and the soil, the modelling of the soil is improved in this work. Similar to Pasternak's assumptions [33], the soil is taken equivalent to a combination of a fictitious layer (shear layer) ensuring the transfer of shear stresses between the sub-base course and Winkler's springs. The all four layers are then modelled by means of the M4-5n.

This paper is based upon Nasser's work presented in [34]. In addition it includes the full and detailed method to construct the "2D plane strain reference case" and the calculus of the elastic stress energy of the M4-5n. In the first section, the specific elastic model denoted finally M4-5nW is presented. In the second section, a composite pavement structure is studied in the aim to determine and to optimize the thickness of the equivalent layer that is added to the Winkler's massif soil. The third section illustrates the advantages of such modelling for a 2D cracked pavement case. Few parametric calculations to determine the most critical load position relative to the existence of a vertical crack and to examine some thermal changing effects in a composite pavement structure are finally illustrated on the distribution of some interface stresses.

2. Development of M4-5nW for the pavement structures

In this section, the Multi-Particle Model of Multilayer Materials (M4-5n) with 5n equilibrium equations (n: total number of layers) is presented. Then the modelling for the soil is discussed in the aim to determine the thickness of the shear layer.

2.1. The M4-5n

2.1.1. General system of the M4-5n equations

The M4-5n is adopted to simulate bending problems in pavement structures [29]. It belongs to the M4 family [21–23]. Assuming that kinematic fields and stress fields may be written per layer rather than per the total thickness of the multi-layered, the M4 family estimates the mechanical fields are thinner than the "Layer Wise plate models" [35]. The M4-5n construction is based on a polynomial approximation by layer in z for the in-plane stress fields (x and y represent the coordinates of the plane of the layers and z represents the vertical coordinate). The thickness of each layer is given by $e^i = h^i_+ - h^i_-$, where h^i_+ and h^i_- are the ordinate of the higher and lower face of layer i respectively (i ∈ {1,n} where n is the total number of layers). The coefficients of these polynomial approximations are expressed via Reissner's classical stress generalized fields. The shear and normal stresses (respectively $\tau_\alpha^{i,i+1}$(x,y) and $\nu^{i,i+1}(x, y)$, $\alpha \in \{1,2\}$, i ∈ {1,n-1}) at the interface between layers i and i+1 (similarly i-1 and i) are ensuring the continuity of the 3D stress field, σ_{kl} (k ∈ {1,3}, l ∈ {1,3}), between these two consecutive layers (Eqs. (1) and (2).

$$\tau_\alpha^{i,i+1}(x,y) = \sigma_{\alpha 3}\left(x,y,h^i_+\right) = \sigma_{\alpha 3}\left(x,y,h^i_-\right) \quad (1)$$

$$\nu^{i,i+1}(x,y) = \sigma_{33}\left(x,y,h^i_+\right) = \sigma_{33}\left(x,y,h^i_-\right) \quad (2)$$

where $\tau_\alpha^{0,1}(x,y)$, $\nu^{0,1}(x, y)$, $\tau_\alpha^{n,n+1}(x,y)$ and $\nu^{n,n+1}(x, y)$ represent respectively the boundary conditions above and below the pavement interface related to interface efforts between the multilayer structure and its external environment.

This model can be viewed as a superimposition of n Reissner plates linked by interfacial forces. Between two adjacent material layers, it becomes possible to express delamination criteria in terms of interfacial forces [26,36]. The evaluation of interface stresses is obtained by a method based on the Hellinger–Reissner variational principle [24]. This formulation reduces then the real problem 3D at the determination of plane fields (x, y) per each layer i and interface i, i+1, (and i−1, i). These fields in the plane (x,y) are regular. Thus the real object 3D (2D) is transforming into one geometry 2D (1D). Furthermore, the M4 approach avoids singularities by giving a finite value of stresses at plate edges [23].

In order to simplify the analysis, the equations of M4-5n used to model the three equivalent pavement layers and the shear layer ensuring the connection between the pavement and the Winkler's soil (Fig. 1), are solved here under the assumption of plane strain conditions and assuming that the volume forces are negligible. Subsequently, the mechanical fields of the M4-5n depend only on variable x.

The layers are numbered from top to bottom of the structure. We denote E^i, e^i and υ^i (i ∈ {1, 4}) respectively the Young's modulus, the thickness, and the Poisson's ratio of each layer i. E^s is the Young's modulus of the soil, and k is the stiffness of the springs. The uniform pressure load is assumed to be applied vertically on the pavement by help to the normal stress $\nu^{0,1}$(x); the shear stress at the interface between the first layer of the pavement and its external environment is null ($\tau_1^{0,1}(x) = 0$).

After combining the mechanical M4-5n equilibrium and behaviour equations per layer i, a system of three 2nd order differ-