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Research paper

Reliability-based design: Artificial neural networks and double-loop reliability-based optimization approaches

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ABSTRACT

Two advanced optimization approaches to solving a reliability-based design problem are presented. The first approach is based on the utilization of an artificial neural network and a small-sample simulation technique. The second approach considers an inverse reliability task as a reliability-based optimization task using a double-loop optimization method based on small-sample simulation. Both techniques utilize Latin hypercube sampling with correlation control. The efficiency of both approaches is tested using three numerical examples of structural design – a cantilever beam, a reinforced concrete slab and a post-tensioned composite bridge. The advantages and disadvantages of the approaches are discussed.

1. Introduction

Tremendous progress has been made in the areas of both reliability and optimization during the last two decades. Reliability-based optimization (RBO), reliability-based design (RBD) and reliability-based design optimization (RBDO) – these terms appear in the literature and represent a combined strategy where we have to deal with the repeated evaluation of an objective function (optimization) and repetitive evaluations of a limit state function (reliability). The concept itself appeared quite early in reliability engineering; see e.g. [1–5]. From those first pioneering works, the concept progressed from reliability-based to risk-based optimization approaches, e.g. [6], emphasizing robustness in structural optimization, e.g. [7,8]. Despite these achievements in the fields of both optimization and reliability, the computational effort required is still enormous for practical problems and we need efficient methods that are easy to apply.

When performing either reliability assessment or engineering design, it is certainly essential to take uncertainties into account using advanced fully probabilistic analysis. Reliability assessment requires forward reliability methods for reliability estimation. On the other hand, engineering design requires an inverse reliability approach in order to determine the design parameters needed to achieve the desired target reliabilities that represent the desired level of reliability in the limit state design of structures. The achievement of such reliabilities is generally not an easy or straightforward task.

Some sophisticated approaches to the determination of design parameters (material properties, geometry, etc.) related to particular limit states have been proposed under the name “inverse reliability methods”, e.g. a reliability contour method [9,10], an iterative algorithm based on the modified Hasofer-Lind-Rackwitz-Fiessler scheme used in reliability analysis [11], the use of a Newton–Raphson iterative algorithm to find multiple design parameters [12,13], the decomposition technique [14] and various implementations of artificial neural networks (ANNs) with other soft-computing techniques [15–17]. The use of ANNs in [1,18] was motivated by the approximate concepts inherent in reliability analysis and the time-consuming repeated analyses required by Monte Carlo type simulation for large-scale structural systems.

The two advanced methods proposed in this paper attempt to overcome the shortcomings of existing inverse reliability methods and are both transparent and relatively easy to apply. Existing inverse reliability methods are generally limited to simple problems and cannot be applied to computationally time consuming problems (such as large finite element computational models). This was the main motivation for the development and software implementation of the techniques presented in this paper. The first method utilizes an ANN too, but in a different way: computational time is reduced by using a small-sample simulation technique called Latin hypercube sampling in an ANN-based inverse problem previously proposed by Novák and Lehký in [19,20].

The second method is the double-loop RBO approach. Classical deterministic optimization usually leads to solutions that lie at

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the boundary of the admissible domain, and that are consequently rather sensitive to uncertainty in the design parameters. In contrast, RBO aims at designing the system in a robust way by minimizing an objective function under reliability constraints. It provides the means for determining the optimal solution for a certain objective function, while ensuring that there is only a predefined small probability that a structure fails. RBO methods thus have to mix optimization algorithms together with reliability calculations. The approach known as "double-loop" consists in nesting the computation of the failure probability with respect to the current design within the optimization loop. A FORM-based double-loop approach has been proposed by Dubourg in [21,22]. The authors of the present paper have developed a double-loop reliability-based optimization approach based on small-sample simulation and the first order reliability method (FORM) [23].

The efficiency of both approaches is tested using three numerical examples of structural design – a cantilever beam, a reinforced concrete slab and a post-tensioned composite bridge. This current paper is based upon Lehký et al. [24], but includes a more detailed theoretical explanation of the small-sample double-loop reliability-based optimization method, including an Aimed Multilevel Sampling strategy for the reduction of sampling space. In addition, an application to a real bridge structure is also included for demonstration purposes and there is a discussion of the practical usability of both approaches.

2. Reliability-based design

2.1. Reliability problem formulation

The aim of classical (forward) reliability analysis is the estimation of unreliability using a reliability indicator called the theoretical failure probability, defined as:

$$p_\mathrm{f} = \mathrm{P}(Z \le 0), \tag{1}$$

where $Z = g(\mathbf{X})$ is a variable called safety margin, which is a function of a random vector, $\mathbf{X} = \{X_1, X_2, \ldots, X_{N\mathrm{var}}\}^\mathrm{T}$, where N_var is the number of random variables. Random vector $\mathbf{X}$ follows a joint probability distribution function (PDF) $f_\mathbf{X}(\mathbf{x})$; in general, its marginal variables can be statistically correlated. The classical approach deals with situations where the information about $f_\mathbf{X}(\mathbf{x})$ is limited to knowledge of univariate marginal distributions $f_{X_1}(x), \ldots, f_{X_{N\mathrm{var}}}(x)$ and a correlation matrix, $\mathbf{T}$ (a symmetric square matrix of order N). The output variable Z represents a transformed variable and the task is to perform reliability analyses upon it. It is assumed that the analytical analysis of the transformation of input variables to Z is not possible due to the complexity of the computational model of $g(\mathbf{X})$. The failure probability is calculated as a probabilistic integral:

$$p_f = \int_{-\infty}^{\infty} I[g(\mathbf{X})] f_\mathbf{X}(\mathbf{x})\mathrm{d}\mathbf{x} = \int_{D_f} f_\mathbf{X}(\mathbf{x})\mathrm{d}\mathbf{x}. \tag{2}$$

The function $I[g(\mathbf{X})]$ is an indicator function that equals one for the failure event ($g(\mathbf{X}) \le 0$) and zero otherwise. In this way, the domain of integration of the joint PDF is limited to the *failure domain* D_f, where $g(\mathbf{X}) \le 0$. The explicit calculation of the integral in (2) is generally impossible. Thus, many efficient stochastic analysis methods have been developed over the last decades, varying in efficiency, accuracy, and suitability for a particular class of problems, as can be tracked from the proceedings of major reliability conferences, e.g. ICOSSAR [25] and ICASP [26]. For practical applications, the calculated failure probability p_f can be substituted by the reliability index $\beta = -\Phi^{-1}(p_\mathrm{f})$ obtained by inverse transformation from the standard normal distribution Φ. This makes it more feasible to solve the reliability problem from the numerical point of view.

2.2. Inverse reliability problem formulation

The design of a structure or part of one in order to achieve a required level of reliability is a typical example of an inverse problem. The aim is to find input design parameters $\mathbf{d} \subseteq \mathbf{X}$ (where $\mathbf{d}$ may contain deterministic design variables and/or statistical parameters of random variables) which yield the corresponding structural safety described by probability indicators – failure probabilities $\mathbf{p}_\mathrm{f}$ or reliability indices $\boldsymbol{\beta}$ – related to different limit states. Then, Eq. (2) for multiple limit states $\mathbf{g}(\mathbf{X})$ can be formally generalized as:

$$\mathbf{p}_\mathrm{f} = \mathbf{F}(\mathbf{X}), \tag{3}$$

where $\mathbf{F}(\mathbf{X})$ is the vector of integral functions according to Eq. (2). Design parameters $\mathbf{d} \subseteq \mathbf{X}$ can be obtained from Eq. (3) via inverse transformation:

$$\mathbf{d} = \mathbf{F}^{-1}(\mathbf{p}_\mathrm{f}). \tag{4}$$

The analytical solution of an inverse problem is usually only possible when using deterministic analysis, and just in simple cases even then. In other cases, a trial-and-error procedure is often carried out in which the estimation of design parameters is performed (mostly based on empirical relationships and/or recommendations), and then the reliability of the system is assessed.

Once we have to deal with the fully probabilistic analysis of a structure, an analytical solution or the utilization of a trial-and-error procedure is time-consuming and inefficient, or even impossible. In such cases, it seems necessary to use advanced methods such as those described in the following sections.

The aim of solving an inverse reliability problem is to find design parameters corresponding to specified reliability levels expressed by reliability indices or by theoretical failure probabilities. In general, an inverse problem involves finding either a single design parameter to achieve a given single reliability constraint or multiple design parameters to meet specified multiple reliability constraints. The design parameters can be deterministic, or they can be associated with random variables described by statistical moments (mean value, standard deviation) and a PDF. In the case of a mean value, one needs to choose if either the standard deviation (absolute variability) or the coefficient of variation (relative variability with respect to the mean) will be fixed.

2.3. Small-sample simulation

A common feature of both approaches presented below is the usage of a small sample simulation technique of the Monte Carlo type. An implementation of this method, called Latin hypercube sampling (LHS), appeared to be the most effective. The main feature of the method is the division of the probability distribution function into non-overlapping intervals of the same probability. Then, the representative values from the intervals (random selection, middle of interval or mean value) are used in the simulation process. The samples are chosen directly from the distribution function based on the inverse transformation of the distribution function. The technique can efficiently cover a multidimensional space of random variables with a small number of simulations [27,28].

The basic feature of LHS is that the range of univariate random variables is divided into N_sim intervals (N_sim is a number of simulations). The values from the intervals are then used in the simulation process (random selection, the median or mean value). The selection of the intervals is performed in such a way that the range of the probability distribution function of each random variable is divided into intervals of equal probability $1/N_\mathrm{sim}$. The samples are chosen directly from the distribution function based on an inverse transformation of the univariate distribution function.