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Research paper

Quasi static and dynamic inelastic buckling and failure of folded-plate structures by a full-energy finite strip method

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ABSTRACT

A study on how a mathematical material modeling approach named rheological-dynamical analogy (RDA) can be used to predict the quasi static and dynamic inelastic buckling and failure of structures is presented in this paper. An analysis of the uniformly compressed folded-plate structures, made of isotropic materials, is carried out. Two sources of non-linearity, one involving geometrical non-linearity due to large deflection, and the other involving material non-linearity due to inelastic behavior, are analyzed by implementing a full-energy finite strip method (FSM). The material non-linearity is analyzed using the RDA. A very basic continuum damage model with one damage parameter is implemented in conjunction with a mathematical material modeling approach in order to address stiffness reduction due to inelastic behavior. According to the analogy, a very complicated material non-linear problem in the inelastic range of strains is solved as a simple linear dynamic one. The orthotropic constitutive relations are derived and modulus iterative method for the solution of nonlinear equations is presented.

1. Introduction

The folded-plate structures are structures which are generally made by joining flat plates at their edges. An important sub-set of these structures, which are the main concern of this paper, are essentially prismatic forms that can have some stiffeners, such as those used in column members, stiffened slabs and box girders. The analysis of the behavior of these structures is done using the finite strip method (FSM). The FSM is based on eigenfunctions, which are derived from the solution of the beam differential equation of transverse vibration, and proved to be an efficient tool for analyzing a great deal of structures for which both geometry and material properties can be considered as constant along a main direction. This method was pioneered by Cheung [1], who combined the plane elasticity and the Kirchhoff plate theory. Wang and Dawe [2] have applied the elastic geometrically nonlinear FSM to the large deflection and post-overall-buckling analysis of diaphragm-supported plate structures.

Current versions of FSM are capable of modeling the plates with different thickness and material properties along the longitudinal direction of the strip. Naderian at al. proposed an integrated finite strip discretization scheme for the flat shell spline finite strip for modeling of long-span cable-stayed bridges [3]. This approach of FSM is a very efficient technique and its application to the dynamic inelastic buckling of thin-walled structures is highly appreciated. Kwon and Hancock [4] developed the spline FSM to handle local, distortional and overall buckling modes in post-buckling range. Interaction of two types of column failure (buckling) in thin-walled structures, local and global (Euler) column buckling, may generate an unstable coupled mode, rendering the structure highly imperfection sensitive.

The geometrically nonlinear harmonic coupled finite strip method (HCFSM) [5,6] is also one of the many procedures that can be applied to analyze the large deflection of thin-walled structures. An analysis of the buckling–mode interaction is carried out using the HCFSM in [7], taking into account the visco-elastic behavior of material. Furthermore, to address these issues, strips with non-uniform characteristics in the longitudinal direction and transverse stiffeners have been used in geometrical non-linear static analysis of prismatic shells [8]. A very important effort for improvement of this approach via parallelization is presented in [9,10].

If uniformly compressed folded-plate structures or thin-walled structures undergo inelastic deformations, these structures generally sustain two sources of non-linearity (geometrical non-linearity due to large deflection and material non-linearity due to inelastic behavior). The analysis presented in this paper is based on the RDA [11,12]. The RDA is a type of inelastic analysis, which transforms

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one category of very complicated material non-linear problems to simpler linear dynamic problems by using modal analysis [13].

In this paper, we present a new approach that combines the RDA and damage mechanics [14] to solve the inelastic buckling problem of folded-plate structures by implementing a full-energy finite strip method (FSM). The uniaxial RDA dynamic modulus is obtained based on a concept of a complex modulus of viscoelasto-plastic (VEP) materials. This modulus is used to obtain one simple continuous modulus function and a stress–strain curves which were verified with experiments on concrete cylinders [15]. When the critical stress exceeds the limit of elasticity, the first iteration of the modulus provides the Hencky loading function and the Von Mises yield stress, whereas the next ones involve the strain-hardening of the material through visco-plastic flow. At the end of these iterations there occurs the failure under compression. The key global parameters, such as the creep coefficient, Poisson's ratio and the damage variable, are functionally related. However, it is a fact that material damage growth is accompanied by an emission of elastic waves which propagate within the bulk of the material [16]. Because of that, a 3D analysis of the propagation of mechanical waves is used to determine the values of undamaged material parameters. The initial properties of steel and aluminum determined on test cylinders and based on longitudinal resonance frequencies [17], are used in the numerical applications. For the analysis of folded-plate structures using the FSM, an inelastic isotropic 2D constitutive matrix is derived starting from the uniaxial state of stress. Although the quasi-static and dynamic constitutive relations are derived for isotropic materials, different stress components induce orthotropy in the material through the RDA modulus-stress dependence. The nonlinear term is the stiffness matrix, which depends on the inelastic orthotropic constitutive matrix. Because of that, a modulus iterative method for the solution of nonlinear equations is presented. In the case of inelastic buckling of rectangular slabs, it has been demonstrated that convergence of the method is fast and that it gives satisfactorily accurate solutions in only several iterations [18]. This paper is based on the conference paper [18], but the research presented in this paper shows that quasi static and dynamic inelastic buckling is not a trivial task for real folded-plate structures and their members. For that reason, this paper also provides possible directions for further studies in the field of folded-plate structures and thin-walled open and closed cross-sections of any shape.

2. FSM for bifurcation buckling and free vibration

2.1. Nonlinear geometric relation

The nonlinear strain–displacement relations in FSM can be predicted by combination of plane elasticity and the Kirchhoff plate theory. This has been accomplished in [5], by using the second-order terms of Green–Lagrange strains. However, since longitudinal loading is assumed here (see Fig. 1), the second-order terms are only necessary for the longitudinal normal strain

$$\varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u_0}{\partial y}\right)^2 + \left(\frac{\partial v_0}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - z\frac{\partial^2 w}{\partial y^2} \quad (1)$$

where u_0 and v_0 are, respectively, displacements in the middle surface in x and y directions, and w is displacement in z direction.

2.2. Bifurcation buckling

In the FSM, which combines elements of the classical Ritz method and the finite element method (FEM), the general form of the displacement function can be written as a product of polynomials and trigonometric functions

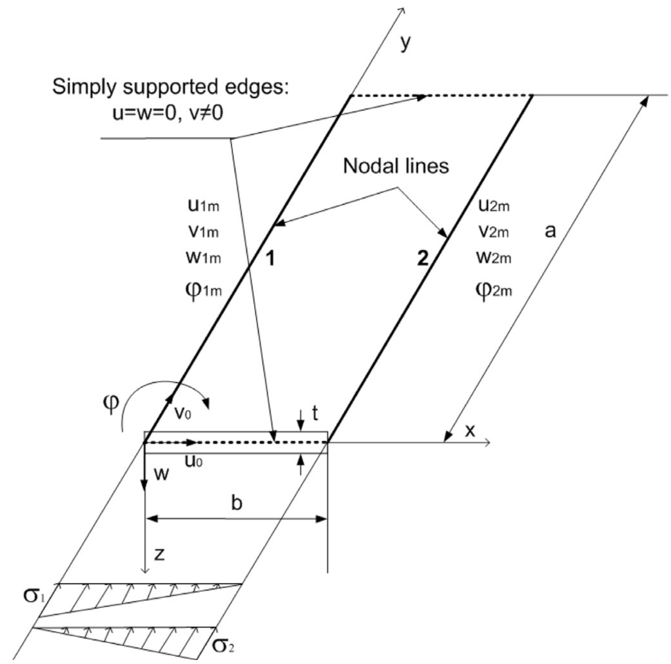

Fig. 1. A simple supported strip with initial stresses.

$$f = \mathbf{Fq} = \sum_{m=1}^{r} Y_m(y) \sum_{k=1}^{c} \mathbf{N}_k(x)\mathbf{q}_{km} \quad (2)$$

where $Y_m(y)$ are functions from the Ritz method and $\mathbf{N}_k(x)$ are interpolation functions from the FEM [1]. We define the local Degrees Of Freedom (DOFs) as the displacements and rotation of a nodal line (DOFs = 4), as shown in Fig. 1. The DOFs are also called generalized coordinates.

The total potential energy of a strip is designated Π and is expressed with respect to the local DOFs

$$\Pi = U + W = (U_m + U_b) + W = \left(1/2\int_A \mathbf{q}_u^T\mathbf{B}_{u1}^T\mathbf{A}\mathbf{B}_{u1}\mathbf{q}_u dA + 1/2\int_A \mathbf{q}_w^T\mathbf{B}_{w3}^T\mathbf{D}\mathbf{B}_{w3}\mathbf{q}_w dA\right) - \int_A \mathbf{q}^T\mathbf{F}^T\mathbf{p}dA. \quad (3)$$

In order to obtain the equilibrium equations, the principle of the stationary potential energy is invoked

$$\frac{\partial \Pi}{\partial \mathbf{q}_m^T} = \mathbf{0}. \quad (4)$$

Eq. (4) gives a linear set of algebraic equations

$$\left(\hat{\mathbf{K}}_{uu}\mathbf{q}_u + \hat{\mathbf{K}}_{ww}\mathbf{q}_w\right) - \mathbf{Q} = \hat{\mathbf{K}}\mathbf{q} - \mathbf{Q} = \mathbf{0}. \quad (5)$$

Well-known elements of the property matrices **A** and **D** for the orthotropic elastic material are

$$K_x = \frac{E_x}{1-\mu_x\mu_y},\ K_y = \frac{E_y}{1-\mu_x\mu_y},\ K_1 = \frac{\mu_y E_x}{1-\mu_x\mu_y} = \frac{\mu_x E_y}{1-\mu_x\mu_y},\ K_{xy} = G$$
$$A_{11} = K_x t,\ A_{22} = K_y t,\ A_{12} = K_1 t,\ A_{66} = K_{xy} t$$
$$D_{11} = K_x\frac{t^3}{12},\ D_{22} = K_y\frac{t^3}{12},\ D_{12} = K_1\frac{t^3}{12},\ D_{66} = K_{xy}\frac{t^3}{12}. \quad (6)$$

Consider the simply supported flat shell strip shown in Fig. 1. The strip is subjected to an initial stress σ, which varies linearly from side 1 to side 2, but is constant along the longitudinal axis

$$\sigma_{ij} = \sigma_{22} = \sigma_y = \left(1 - \frac{x}{b}\right)\sigma_1 + \frac{x}{b}\sigma_2. \quad (7)$$