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Research paper

Noise attenuation capacity of a Helmholtz resonator

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A R T I C L E I N F O

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A B S T R A C T

Helmholtz resonator (HR) is one of the most basic acoustic models and has been widely used in engineering applications due to its simple, tunable and durable characteristics. The transmission loss index is mainly used to evaluate the acoustic transmission performance. Based on the transmission loss index, this paper proposes the noise attenuation capacity index as one of the key parameters to evaluate the noise attenuation performance of a HR. The noise attenuation capacity is defined as the integral of transmission loss in the frequency domain. The theoretical formula of a HR's noise attenuation capacity is first derived in this study. It indicates that the noise attenuation capacity of a HR is only related to geometries of the neck and duct's cross-sectional area. The cavity volume has no effects on its noise attenuation capacity. The proposed theoretical formula of a HR's noise attenuation capacity is validated by Finite Element Method (FEM) simulation using commercial software (COMSOL Multiphysics). The proposed noise attenuation capacity of a HR should therefore be considered as one of the main acoustic characteristics of a HR. It is hoped that the present study could provide a stepping stone for the investigation of the HR's or other silencers' noise attenuation capacity and potential applications in all research areas in respect of the HR.

1. Introduction

The Helmholtz resonator (HR), which consists of a cavity communicating with an external duct through an orifice, is a well-known device to reduce noise centralized in a narrow band at its resonance frequency. Owing to the resonance frequency of a HR is only determined by its geometries, it is therefore straightforward to obtain a HR with a desired resonance frequency [1,2]. It is because of its simple, tunable and durable characteristics, the HR has been utilized in numerous duct-structure systems, such as ventilation and air conditioning system in buildings, automotive duct systems and aero-engines, for the attenuation of noise produced by unavoidable in-ducted elements [3,4]. Moreover, the applications of HRs extend to other research areas, for instance notch filters [5] and ultrasonic metamaterials [6].

Since the widespread applications of the HR, it has received a great deal of attentions worldwide. A lot of achievements have been made and are documented in numerous pieces of literature. Many studies have tried to obtain an accurate prediction of the resonance frequency. Initially, the HR is regarded as an equivalent spring-mass system. The mass of air in the neck is driven by an external force and the air inside the cavity acts as a spring [7]. Furthermore, wave propagation in both the neck and cavity has been considered in theoretical analysis. The wave propagation approach has developed from a one-dimensional approach in preliminary investigations to a multidimensional approach for the sake of accuracy [8,9]. Because of the HR is one of the most basic acoustics models as well as the narrow-band behavior at its resonance frequency, a wealth of literature also exists on the modification forms of HRs in order to improve the acoustic performance of a HR. The effects of different orifices and cavity geometries on the acoustics performance have been studied [10]. Besides, some novel HRs have been proposed and investigated, for instance HR with extended neck or spiral neck [11], dual HR [12], coupled HR [13,14], HR with a coiled air cavity [15] and micro-perforated panel absorbers backed by HR (MPPHR) [16]. The major concerns of these modification forms of HRs are related to the transmission loss performance in the frequency domain.

The transmission loss index is indeed a major index and has been widely used to assess the acoustic transmission performance in the frequency domain. However, almost all researches concentrate on the shapes of the transmission loss curve while ignoring the area under the transmission loss curve. The noise attenuation capacity index defined as the integral of transmission loss in the frequency domain is therefore proposed to be one of the key parameters to evaluate HR's noise attenuation performance. The theoretical formula of a HR's noise attenuation capacity is first derived in this paper. Then, the three-dimensional Finite Element Method simulation using commercial software (COMSOL Multiphysics [17]) is adopted to verify the correctness of the proposed theoretical formula. The proposed noise

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attenuation capacity should be considered as one of the main acoustic characteristics of a HR. It hopes that the present study could provide a stepping stone for the investigation of the HR's or other silencers' noise attenuation capacity and potential applications in all research areas in respect of the HR.

2. Theoretical analysis of the noise attenuation capacity of a Helmholtz resonator

The sudden discontinuous areas, for instance the duct-neck interface and the neck-cavity interface, result in a clearly multidimensional sound fields inside a HR [18]. Although a multidimensional approach could provide a more accurate prediction of the acoustic impedance of a HR, the main purpose here is to reveal the HR's noise attenuation capacity. Moreover, the dimensions of the traditional HRs are significant small compared to the wavelengths of the concerned low frequencies in this study. It is therefore that classical equivalent spring-mass system is adopted here by introducing an end correction factor to account for the effects of evanescent high-order modes.

2.1. The classical lumped approach of a Helmholtz resonator

For the sake of completeness, a brief review of the classical lumped approach of a HR is appropriate here. A mechanical analogy of a single HR is illustrated in Fig. 1. The mass of air in the neck $M_m = \rho_0 S_n l_n'$ is driven by an external time-harmonic sound pressure force $F = S_n p_0 e^{j\omega t}$ and the cavity is regarded as a massless spring with stiffness $K_m = \rho_0 c_0^2 S_n^2/V_c$ (where p_0 is the oscillation sound pressure, ρ_0 is air density, c_0 is the speed of sound in the air, l'_n and S_n are the neck's effective length and area respectively, ω is the angular frequency, and V_c is the cavity volume). The damping coefficient R_m of a HR is mainly caused by viscous dissipation through the neck, which is determined by acoustic screen across the area of the neck. By applying the Newton's second law of motion to the one degree of freedom HR, the oscillatory differential equation can be expressed as [1]:

$$M_m \frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + K_m x = S_n p_0 e^{j\omega t} \tag{1}$$

where x is the displacement of the mass, $v = dx/dt$ represents the velocity of the mass.

Owing to the different concerns between an acoustic system and a mechanical system, Eq. (1) should be rewritten in the form of volume velocity $U = vS_n$ as:

$$M_a \frac{dU}{dt} + R_a U + C_a \int U dt = p_0 e^{j\omega t} \tag{2}$$

where $M_a = M_m/{S_n}^2$, $R_a = R_m/{S_n}^2$ and $C_a = {S_n}^2/K_m$ represent the sound mass, sound resistance and sound capacitance respectively in analogy of a circuit. The impedance of the HR can be derived from the solution of Eq. (2) as:

$$Z_r = \frac{p}{U} = R_a + j\left(\omega M_a - \frac{1}{\omega C_a}\right) \tag{3}$$

It is therefore that the resonance frequency of the HR can be derived from Eq. (3) and be expressed as $f = \sqrt{1/M_a C_a}/2\pi = c_0\sqrt{S_n/l_n' V_c}/2\pi$. Once the impedance is obtained, the transmission loss of a side-branch HR mounted on a duct with cross-sectional area S_d can be expressed as:

$$TL = 20\log_{10}\left(\frac{1}{2}\left|2 + \frac{\rho_0 c_0}{S_d}\frac{1}{Z_r}\right|\right) \tag{4}$$

2.2. Noise attenuation capacity of a Helmholtz resonator

The transmission loss index is mainly used to evaluate the acoustic transmission performance in the frequency domain. However, it cannot provide a quantitative characteristic of the noise attenuation band. It is therefore that this paper proposes the noise attenuation capacity index as one of the key parameters to evaluate the HR's noise attenuation performance quantitatively and distinctly. The noise attenuation capacity C_{TL} which is defined as the integral of transmission loss in the frequency domain, could be expressed as:

$$C_{TL} = \int TLdf = \frac{1}{2\pi}\int TLd\omega = \frac{1}{2\pi}\int 20\log_{10}\left(\frac{1}{2}\left|2 + \frac{\rho_0 c_0}{S_d}\frac{1}{Z_r}\right|\right)d\omega \tag{5}$$

The effect of viscous dissipation through the neck is ignored for simplicity. It is therefore that R_a in Eq. (3) equals zero. Then, substituting Eq. (3) into Eq. (5) gives:

$$\int TLd\omega = \int 10\log_{10}((B\omega^2 - C)^2 + A^2\omega^2)d\omega - \int 10\log_{10}(B\omega^2 - C)^2 d\omega \tag{6}$$

where $A = \rho_0 c_0/2S_d$, $B = \rho_0 l'_n/S_n$ and $C = \rho_0 {c_0}^2/V_c$ are constants related to geometries of the HR and the duct. The antiderivative of the first term on the right-hand side of Eq. (6) can be solved as:

$$\begin{aligned}\int 10\log_{10}((B\omega^2 - C)^2 + A^2\omega^2)d\omega &= \int 10\log_{10}(B^2(\omega^2 + a)(\omega^2 + b))d\omega \\ &= \int 10\log_{10}B^2 d\omega + \int 10\log_{10}(\omega^2 + a)d\omega + \int 10\log_{10}(\omega^2 + b)d\omega\end{aligned} \tag{7}$$

and

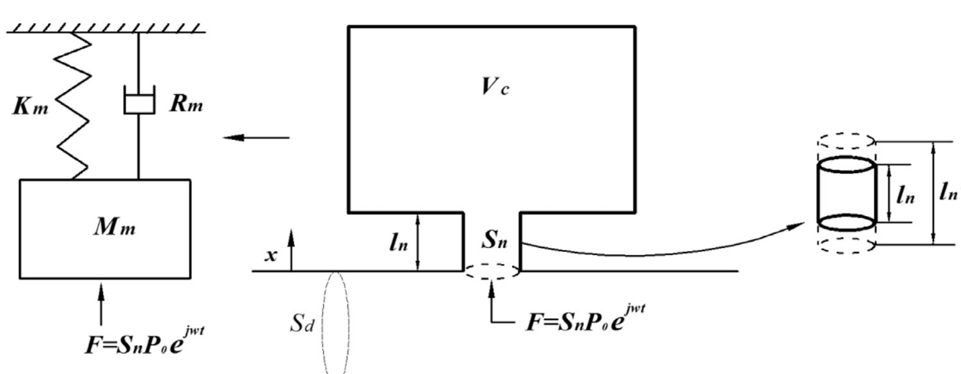

Fig. 1. Mechanical analogy of a Helmholtz resonator.