ARTICLE IN PRESS

Advances in Engineering Software xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Research paper

Least-Square-Support-Vector-Machine-based approach to obtain displacement from measured acceleration

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ARTICLE INFO

Keywords:
Least square support vector machine
Accelerogram processing
Displacement signal
Kernel regression

ABSTRACT

Recent advances in computer and sensing technologies have led to the proliferation of sensor networks in structural health monitoring and condition monitoring applications.

Vibration data collected by sensors provide useful information about the condition of a structure or a machine component, facilitating identification of any changes in its performance. While acceleration and displacement data provide complementary information, a cost-effective alternative to monitoring both is to estimate displacements from accelerations. This paper presents a kernel regression approach for obtaining displacement time series from acceleration data. Starting from a second-order central difference approximation, the method performs ridge regression in a feature space induced by the linear kernel. The main advantages of the proposed method are (1) It does not require baseline adjustment, other than removing the mean of the acceleration record; (2) The solution obtained is numerically stable, and thus regularization is not necessary; (3) The reconstructed displacement does not exhibit any long period drift. The validity of the proposed method is demonstrated through examples, where structural systems' displacements computed using the proposed approach were compared to the recorded experimental displacements. While the presented examples focus only on monitoring of vibrations responses of structural systems, the proposed method can be used in other settings where a displacement signal is to be estimated from an acceleration signal with appropriate, application-specific modifications.

1. Introduction

With recent advances in sensing technology and wireless communication, the last few decades have seen a dramatic increase in the use of sensor networks for structural health monitoring and condition monitoring applications. Analysis of sensor data provides useful information about the condition of a structure or a machine component, enabling engineers and inspectors to identify and monitor any changes in its performance.

Estimation of a displacement time series from an acceleration record is an important task in vibration monitoring. While it is possible to measure displacements using, for example, the linear variable differential transformer, direct measurement of displacement is often inconvenient and costlier than measurement of acceleration, especially when displacements at different points on the structure or the machine are needed. Estimation of displacements from accelerations is a cost-effective alternative to direct measurement.

While the mathematical relationship between displacement and acceleration is simple, problems arise in practical applications. Exact computation of displacement from acceleration requires not only the closed-form equation of the acceleration expressed as a function of time but also the initial values of the velocity and displacement. In practice, acceleration data is often available in discrete form (digitally recorded, or digitized from analog records), and the initial displacement and velocity are unknown.

A common concern when a displacement history is to be obtained using numerical integration, is the accumulation of error with each integration step. Numerical integration will amplify low-frequency noise in the acceleration signal, resulting in low-frequency drifts in the estimated displacement. While large distortions can be eliminated using low-cut filtering, given the sensitivity of the estimated displacements to the specifics of the filter used, proper selection of the low-cut filter and its associated parameters remains a critical issue. The same conclusion can be extended to baseline adjustment techniques, which effectively perform low-cut filtering, although the frequency domain characteristics of the corresponding filter may be implicit.

Hong et al. [1] formulated a new approach for estimating the displacement signal from the acceleration signal by converting the initial

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http://dx.doi.org/10.1016/j.advengsoft.2017.10.011
Received 27 July 2017; Received in revised form 28 September 2017; Accepted 18 October 2017
0965-9978/

Please cite this article as: Tezcan, J., Advances in Engineering Software (2017), http://dx.doi.org/10.1016/j.advengsoft.2017.10.011

value problem to a boundary value problem. This results in an inverse problem which is ill-posed and thus does not have a unique solution. Using Tikhonov regularization, they make the boundary value problem well-posed, making it admit a unique solution. An additional potential benefit of regularization is to reduce the effect of measurement noise in the acceleration signal, by encoding a preference toward smaller displacement values.

In this paper, we propose a kernel-based alternative to the regularization scheme derived by Hong et al. The proposed method uses Least Square Support Vector Machine (LSSVM) to solve the problem in a kernel-induced feature space, eliminating the issue of rank deficiency, thereby ensuring the uniqueness of solution without the necessity of regularization.

The rest of the paper is organized as follows. Section 2 briefly discusses the problem of displacement reconstruction from measured acceleration data, recasts the problem as an ill-posed boundary value problem, and shows that the resulting problem can be solved using Tikhonov regularization, as suggested by Hong et al. [1]. Section 3 introduces the Least Square Support Vector Machine (LSSVM) approach to the same problem. In our formulation, the solution is obtained in the feature space without the need for regularization. While regularization is not needed to correct the ill-posedness of the problem, we include an optional regularization factor which allows performing regularization for reasons not related to numerical stability, i.e. when it is suspected that the acceleration data contains measurement noise. Section 4 demonstrates applications of the proposed method on monitoring vibrations of structural systems. Section 5 presents a discussion of the limitations of the method. Using challenges specific to processing of strong motion data as an example, it is stressed that successful application of the proposed method to different problems may require additional steps, depending on the specifics of the acceleration signal and the goal of processing. Section 6 concludes the paper.

2. Estimation of displacement from measured acceleration

This section reviews two existing approaches to the problem of displacement reconstruction from a measured acceleration record. The equations below use the following notation: Vectors are shown in bold italic, matrices in bold, and scalars in italic. By default, vectors are column vectors, and the superscript T denotes transpose operation.

Given a discretized acceleration signal $\boldsymbol{a} = [a_1, a_2, \ldots, a_N]^{\mathrm{T}}$, the corresponding velocity signal $\boldsymbol{v} = [v_1, v_2, \ldots, v_N]^{\mathrm{T}}$ and the displacement signal $\boldsymbol{d} = [d_1, d_2, \ldots, d_N]$ can be estimated using numerical integration, provided that initial conditions are known. For example, Newmark-β method uses

$$\hat{v}_i = \hat{v}_{i-1} + \Delta t((1-\gamma)a_{i-1} + \gamma a_i),$$
$$\hat{d}_i = \hat{d}_{i-1} + \Delta t\ \hat{v}_{i-1} + ((0.5-\beta)a_{i-1} + \beta a_i)(\Delta t)^2, \tag{1}$$

where $\hat{v}_i$ and $\hat{d}_i$ are the estimated velocity and displacement at the *ith* time step, β and γ are integration parameters corresponding to the assumed manner of variation of acceleration, and Δt is the time step.

Numerical integration often fails to produce realistic displacement estimates. A common problem is the low-frequency drift in the resulting displacement history. Fig. 1 illustrates the drift problem which occurs when the displacement history is obtained via double integration of acceleration. The experimental data in Fig. 1 corresponds to the acceleration and displacement responses at the second floor of a forced vibration test of a two-story reinforced concrete frame. A horizontal input excitation was applied at the top of the frame through a linear inertial shaker [2].

To date, various accelerogram processing procedures have been proposed to obtain realistic displacement estimates (e.g. [3–6]). The displacement reconstruction task can also be viewed as a boundary value problem. Consider the problem of estimating acceleration from a measured displacement signal $\boldsymbol{d}$ using the second-order central

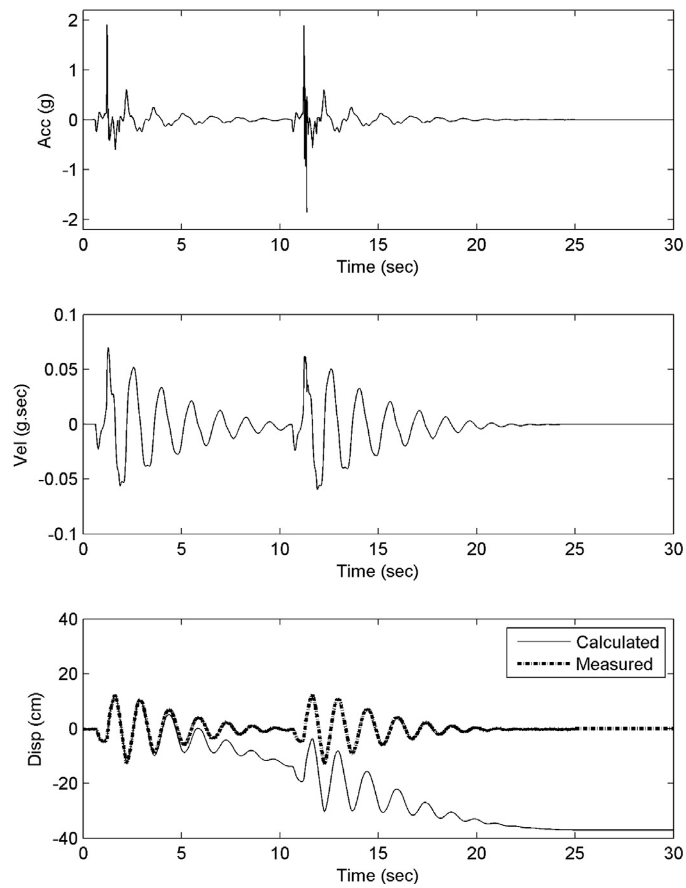

Fig. 1. Comparison of measured displacement and the calculated displacement obtained via double integration of acceleration.

difference approximation $\hat{a}_i = (d_{i-1} - 2d_i + d_{i+1})/(\Delta t)^2 + E$ where E is the local truncation error $O((\Delta t)^2)$. The corresponding matrix equation for the estimation of an m-term segment of an acceleration signal can be written as $\mathbf{B}\boldsymbol{d} = (\Delta t)^2\hat{\boldsymbol{a}}$, where $\hat{\boldsymbol{a}} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_m]^{\mathrm{T}}$ is the estimated acceleration vector within an m-point time window $[t_1, t_2, \ldots, t_m]$, and $\mathbf{B}$ is the $m \times (m+2)$ matrix

$$\boldsymbol{B} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \cdots & & & & \\ 0 & 1 & -2 & 1 & 0 & & & & & \\ & 0 & 1 & -2 & 1 & 0 & & & & \\ & & & \ddots & \ddots & \ddots & & & & \\ & & & & \ddots & \ddots & \ddots & & & \\ & & & & & \ddots & -2 & 1 & & \\ & & & & & & 1 & -2 & 1 & 0 \\ & & & & & & 0 & 1 & -2 & 1 \end{bmatrix}. \tag{2}$$

Thus, the m-term acceleration vector can be computed uniquely from the (m + 2)-term displacement vector via the matrix equation

$$\begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \cdots & & & & \\ & 1 & -2 & 1 & 0 & & & & & \\ & 0 & 1 & -2 & 1 & 0 & & & & \\ & & & \ddots & \ddots & \ddots & & & & \\ & & & & \ddots & \ddots & \ddots & & & \\ & & & & & \ddots & -2 & 1 & & \\ & & & & & & 1 & -2 & 1 & 0 \\ & & & & & & 0 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_{m-1} \\ d_m \\ d_{m+1} \end{bmatrix} = (\Delta t)^2 \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \vdots \\ \vdots \\ \hat{a}_{m-2} \\ \hat{a}_{m-1} \\ \hat{a}_m \end{bmatrix}. \tag{3}$$

The terms d_0 and d_{m+1} in the displacement vector falling outside the time window act as fictitious nodes that enable computation of the acceleration at the boundaries of the time window, i.e. at $t = t_1$ and at $t = t_m$.