Advances in Engineering Software 000 (2017) 1–10

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

An algorithm for decomposition of sub-domains and quadrilateral mesh generation with line constraints

X. Ma[a,*], G. Zhao[a], D. Zhang[b]

[a] Key Laboratory for Liquid-Solid Structural Evolution and Processing of Materials (Ministry of Education), Shandong University, Jinan 250061, China
[b] Department of Water Hazard Research, China Institute of Water Resource and Hydropower Research, Beijing 100038, China

ARTICLE INFO

Article history:
Received 14 November 2016
Revised 24 April 2017
Accepted 7 May 2017
Available online xxx

Keywords:
Mesh generation
Quadrilateral mesh
Line constraint
Sub-domain decomposition

ABSTRACT

The technique for quadrilateral mesh generation on arbitrary 2-D domain has been quite mature; however, the existing methods and software can not directly deal with mesh generation with internal line constraints. In flood analysis and other analysis, the analysis models often contain a large number of constraint lines. In mesh generation, the sides of elements besides constraint lines must be attached to constraint lines, so that boundary conditions can be defined on these constraint lines. In this paper, a general method for automatic quadrilateral mesh generation with internal line constraints is presented. The mesh generation process is mainly divided into two stages, i.e. the automatic decomposition of sub-domains and quadrilateral mesh generation in each sub-domain. The sub-domains enclosed by constraint lines and boundaries are determined automatically with the proposal algorithm in this paper. Then nodes are generated on each edge of sub-domains based on mesh size requirements. By solving an integer linear programming problem, the number of nodes on each edge is adjusted automatically, so that the total number of boundary nodes in each sub-domain is even, satisfying the necessary condition for generating all-quadrilateral mesh. For free constraint lines in each sub-domain, the method of regarding constraint lines as holes with zero area is proposed, so that constraint lines can be treated as internal boundaries. The merging method of constraint lines and outer boundary is given, which can deal with all possible distribution of constraint lines and has generality. The examples of mesh generation with a large number of constraint lines are given to demonstrate the reliability of proposed method.

1. Introduction

The finite element method, finite volume method and finite difference method are the most widely used numerical methods in science and engineering computation. One of important steps in the application of these numerical methods is to discrete the computational domain, i.e. the mesh generation. Triangular or quadrilateral meshes are usually used in 2-D domain, and due to better computational accuracy and convergence, quadrilateral meshes are preferred in 2-D numerical analysis. In the early application of numerical analysis, the triangular mesh is usually used, which is because the technique for quadrilateral mesh generation is not mature at that time. After decades of development, the algorithm and technique for quadrilateral mesh generation has been quite mature. Corresponding to 2-D domain, 3-D computational domain can be discretized by tetrahedral or hexahedral mesh. At present, the technique for tetrahedral mesh generation has been mature, but the generation of hexahedral mesh on arbitrary 3-D domain is not mature enough, and there are still many problems to be solved [1,2].

Though practical problems to be carried out with numerical analysis are all 3-D problems, a lot of analysis can be simplified as 2-D problems in order to reduce the difficulty of solving and improve the efficiency of solving on the premise of ensuring the accuracy of the solution, for example, the flood analysis. In the analysis of flood routing, because the flood wave has characteristics of steep water level and violent change of flow pattern, special requirements are needed for numerical computation schemes. At present, Godunov scheme has been successfully applied to shallow water simulation because of its ability to simulate large gradient flow and automatically capture shock wave [3]. So, 2-D model with Godunov scheme can be adopted in flood routing analysis. The computational domain in flood routing analysis is quite complex, in which there are various internal wading structures, such as roads, railways, dams, buildings etc. The internal boundary condition needs to be defined along these structures in analysis model. For example, Fig. 1 shows the computational domain and internal wading structures for urban flood analysis of Dongguan City, China.

* Corresponding author.
E-mail address: maxinwu@sdu.edu.cn (X. Ma).

http://dx.doi.org/10.1016/j.advengsoft.2017.05.004
0965-9978/

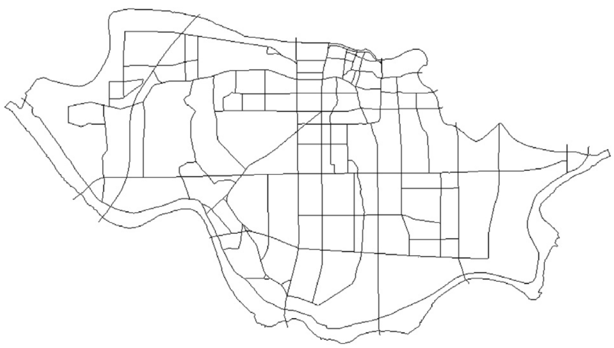

Fig. 1. Computational domain and wading structures.

The internal wading structures should be considered as constraint lines when mesh generation in computational domain. The nodes should be generated along constraint lines and the sides of elements besides constraint lines should be attached to the constraint lines.

The mesh generation for above domain with internal constraint lines cannot be realized automatically utilizing commercial software available. It needs user to divide computational domain that contains constraint lines into sub-domains manually and add auxiliary lines if necessary. Thus, the constraint lines will constitute boundaries of sub-domains. The nodes are generated along external boundary and constraint lines and quadrilateral mesh is generated in each sub-domain. The elements in all sub-domains will form the final mesh of computational domain. When there are a lot of constraint lines in computational domain, it will be a tedious work to generate mesh by manual processing method. It causes low efficiency and sometimes the quality of the mesh cannot be guaranteed. For the mesh generation in domains with a large number of constraint lines, an automatic processing method is needed to realize the decomposition of sub-domains and the mesh generation with internal constraint lines.

2. Related work

Quadrilateral mesh generation can be grouped into structured mesh generation and unstructured mesh generation. In structured mesh, all internal nodes have the same number of adjacent elements. The structured mesh can be generated by mapping or sub-mapping method [4,5]. The efficiency of structured mesh generation is very high and the quality of the mesh is also good. But the biggest disadvantage of the method is that it is only suitable for specified configuration and cannot generate structured mesh on arbitrary domain. Corresponding to structured mesh, the internal nodes in unstructured mesh may have not same number of adjacent elements. In order to generate mesh on arbitrary domain, various methods of unstructured mesh generation have been put forward and successfully applied to commercial softwares.

At present, the typical methods for unstructured quadrilateral mesh generation include grid-based algorithm [6–8], Looping algorithm [9–15], Paving algorithm [16–19] and Q-Morph algorithm [20], etc. The complexity of these algorithms is different, and the quality and efficiency of mesh generation is also different.

The grid-based algorithm for quadrilateral mesh generation is relatively simple. It firstly puts regular grids over target domain and removes elements outside target domain, then adjusts or cuts the elements that intersect boundary, and finally through mesh smoothing, the quadrilateral mesh can be obtained. The grid-based algorithm can produce regular elements inside the domain, but the generated mesh is related to initial grid and its orientation, and the quality of boundary elements is relatively poor [6–8]. The Looping algorithm is also known as the "bisection algorithm". It firstly generates nodes along the boundary of the domain. By choosing a proper splitting line, the domain is decomposed into two sub-domains. Then the nodes are created along the splitting line. The recursive decomposition is carried out for each sub-domain until the final sub-domain contains only four or six nodes. After completion of decomposition, the closure is done on each sub-domain that has six nodes and final mesh is got [9–15]. Looping algorithm generates quadrilateral mesh by the way of recursive decomposition, which is very reliable and has been applied to commercial software I-DEAS. Paving algorithm can produce high quality quadrilateral mesh and it may be the most complicated algorithm in unstructured mesh generation. The algorithm is similar to advancing front algorithm in triangular mesh generation. Starting from boundaries, it lays quadrilateral elements inwards layer by layer. The main operations of the algorithm include the generation of elements, sewing, intersection processing, local adjustment and smoothing, etc. Among these operations, intersection processing is a very time consuming and error prone operation. Due to precision error of floating number, intersection checking may fail, which can lead to mesh overlapping and the failure of mesh generation. In addition, when the mesh size has large difference on the fronts, the intersection processing may fail, which can also lead to the failure of mesh generation [16–19]. Currently Paving algorithm has been applied to commercial softwares, such as Patran, Fluent, etc. The above mesh generation algorithms are direct methods, that is, they generate quadrilateral mesh directly in the domain. Q-Morph algorithm is an indirect method to generate quadrilateral mesh, which first generates a triangular mesh in the domain, and then takes a series of operations to merge triangular mesh into quadrilateral mesh. Most operations in Q-Morph algorithm are borrowed from Paving algorithm. But because the operations are carried out over the triangular mesh, the intersection checking and processing is not needed, which make this algorithm more reliable [20]. Q-Morph algorithm has been applied to commercial software Ansys.

Although there are many researches on quadrilateral mesh generation, the literatures about mesh generation with internal constraint lines are very few. Lee et al. [21] proposed a method of quadrilateral mesh with internal constraint lines, which was used to generate mesh for the plate in a ship structure with stiffeners. In the method, the constrained Delaunay method was used to generate triangular mesh firstly in the domain to satisfy line constraints. Then the triangular mesh was transformed into quadrilateral mesh by Q-Morph algorithm. The proposed method can only deal with separate constraint lines and it is not suitable for the condition that the constraint lines have intersections. Park et al. [22] presented an automated scheme of quadrilateral mesh generation for randomly distributed line constraints. The scheme can deal with any intersections of constraint lines and generate quadrilateral mesh by Paving algorithm. The main improvement of the scheme is to construct loops according to distributing patterns of constraint lines and generate quadrilateral mesh on the loops. As the drawback of Paving algorithm, the scheme further increases the complexity of intersection processing, may resulting in the failure of intersection processing and incorrect mesh generation.

In this paper, an automatic method for the decomposition of sub-domains and quadrilateral mesh generation with internal constraint lines is proposed for mesh generation in flood analysis. There are mainly two steps for the mesh generation with internal constraint lines: (1) The sub-domains enclosed by constraint lines and boundaries are decomposed and the boundaries and remaining constraint lines in each sub-domain are determined; (2) The nodes are generated along boundaries and constraint lines and quadrilateral meshes are generated in each sub-domain by Looping algorithm. The meshes on all sub-domains are merged to form the final mesh. It should be noted that if there are no constraint lines in a