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Optimum topology design of multi-material structures with non-spurious buckling constraints

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ABSTRACT

This study contributes an investigation and a removal of spurious buckling modes which occur in optimal multi-material topology designs of steel composite using MATLAB computational software version 2017a. Using multiple steel materials within a structure provides many design possibilities to improve structural stability, especially, to prevent buckling phenomenon of the steel structure. Buckling effect and volume control are stated as constraints in the minimized compliance multi-material topology optimization problem. The multi-phase problem in this study is solved using an alternative active-phase algorithm with Jacobi version. The Method of Moving Asymptotes (MMA) is used to update the topology design variables which is relative element densities. The advantage of MATLAB software in solving a large number of routines problems which are eigenvalue and multi-material calculation routines and the MMA complex optimizer is shown through the numerical analysis in this study. Two material interpolation schemes controlling spurious buckling modes for the single material are modified for the present multi-material problem. The efficiency of each modified scheme for single and multiple steel materials are investigated and verified in numerical implementation.

1. Introduction

When compressive stresses occur in slender steel structure [1,2], the buckling consideration should be a significant concern in safety design. Slenderness often occurs in topology optimization design when prescribed volume fraction is small. Therefore, not only volume but also stability requirement is also considered as a constraint in topology optimization problem.

A specific numerical situation in computing topology optimization with buckling eigenproblem is the appearance of eigenmodes [3] in void regions. These eigenmodes are called pseudo-modes, localized modes or spurious modes [4]. Void regions are not material free but are assigned a very small value of material due to the numerical singularity. This is the reason to cause spurious modes since they are not only originated in material regions but also in void regions. This problem can occur either in stability analysis or eigenfrequency analysis when first buckling load factor or eigenfrequency is declared as an objective function or constraints [5–7]. In topology optimization, instabilities may occur since geometrical stiffness is penalized insufficiently. This problem would be avoided by appropriately interpolating the elasticity properties of structural and geometric stiffness [8].

SIMP interpolation schemes [9] are well suited to solve stiffness optimization problems. However, applying these schemes on eigenvalue optimization problems may result in spurious modes. These usually appear as highly localized modes in low-density regions of the structure during the optimization process. For SIMP schemes, it happens because the generalized density goes to zero, thus gives rise to low eigenvalues.

To obtain an optimal design satisfying design concept of real structures, the buckling modes appearing in low-density regions make no sense, thus they should be ignored from the topology design. The real buckling mode occurring in the material region or high relative density region should be only considered. For the purpose of ignoring buckling modes in low-density regions, different material interpolation schemes are used for elasticity tensor of physical stiffness matrix ($\mathbf{E}_e^{\mathbf{K}}$) and stress stiffness matrix ($\mathbf{E}_e^{\mathbf{K}_\sigma}$).

For one material structure, the common interpolation scheme Solid Isotropic Material with Penalization (SIMP) [10] based on power law is used. This scheme is unwieldy with additional materials. Some interpolation schemes were proposed for multi-material problem such as the extensions two materials with and without void are introduced in [9] or an interpolation scheme of multi-material using direct generalizations of SIMP is formulated within a unified parametrization [11]. However, in those studies, there is no investigation of multi-material interpolation scheme in dealing with spurious buckling modes. In this study, two inter-

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polation schemes removing spurious buckling modes of the single material problem are modified for the present multi-material problem. In order to deal with the multiphase problem [12–14], an active-phase algorithm [15] using Jacobi version is utilized in this study.

The computational processes in this study are executed in MATLAB program, version 2017a which are a high-level programming language and interactive environment software. With the versatility and user convenience, it has been using widely for academic and research purposes [16]. In the field of topology optimization method which has apparent complexity, MATLAB well solves many different complex problems with very few lines of code [15,17–19]. Especially, its big advantage is presented through solving the large number of routines like eigenvalue problem, complex solvers [9] which are main characters of this study. The code used in this study is developed based on the 115-line code [15] to solve the buckling problem using multi-material topology optimization [20,21].

In this study, a steel structure with multi-steel materials is considered. For simplicity, the multi-steel materials are expressed by the difference ratio between each steel type [22]. The properties of material are described through Young's modulus and the Poisson's ratio of $\nu = 0.3$. In the application of particular structures [23,24], the combination of constructional steel materials can be applied for a more reliable investigation [25,26]. The present structure is discretized and analyzed using the four-node square finite elements. With its simple application and acceptable result with coarse mesh for a simple-square structure in this study, it is chosen for analyzing the structure to save the computational effort. However, for more complicated structure, other approaches and methods can be applied in the further researches to enhance the analysis ability of presented method such as isogeometric analysis [27–32], adaptive technique based on ES-FEM and polygonal elements [33–35] or cell-based smoothed finite element method [36–39].

This study consists of as follow. Section 2 shows the formulations of multi-material topology optimization. Optimization solver of Method of Moving Asymptotes (MMA) and active-phase algorithm using Jacobi version are presented. In Section 3, formulations of linear elastic buckling problem are given. The filtering method to avoid checkboard phenomenon is presented in Section 4. Section 5 describes the computational procedure of present problem based on MATLAB program. Two modified multi-material schemes of removing spurious buckling modes are shown in Section 6. Numerical examples of using the two modified schemes for two and three material cases are discussed in Section 7. In Section 8, finally, conclusions and remarks are described.

2. Formulations of multi-material topology optimization problem

2.1. Multi-phase topology optimization

We consider the minimum compliance multi-material topology optimization problem within a fixed two-dimensional design domain $\Omega \in \mathbb{R}^2$ which is divided into a finite element subdomainΩ_j, $j = 1, 2, \ldots, n$. Each subdomain can be materials or void as shown in design space schematic of multi-material topology optimization which is described in Fig. 1. With Ω_s^m and $\Omega_v^m$are solid design domain of material and void domain in a multi-material problem, respectively. m denotes the number of multi-material.

Element densities are set as design variables which can physically attain integer values, i.e., $\alpha_j \in \{0, 1\}$. A single element may contain multiple material densities corresponding to a number of contributed materials p. Volume fraction and buckling load factor

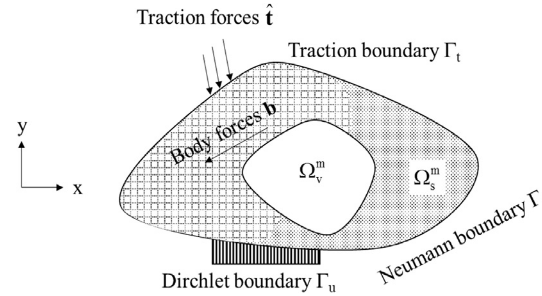

Fig. 1. Design space schematic of multi-material topology optimization.

are stated as inequality constraints. Loads are assumed to be quasi-static and applied to the non-restrained nodes. The mathematical formulation of problem considering buckling effect is as follows

$$
\begin{aligned}
&\underset{\alpha_i}{minimize}: \quad C\,(\boldsymbol{\alpha}, \mathbf{u}) = \mathbf{u}^T\mathbf{K}\mathbf{u} \\
&subject\ to: \quad \mathbf{K}(\boldsymbol{\alpha})\mathbf{u} = \mathbf{f} \\
&\qquad \sum_{i=1}^{p} \alpha_i = 1 \\
&\qquad \int_{\Omega} \alpha_i dx \le V_i, \qquad (i = 1, \ldots, p) \\
&\qquad \min_{j \in J} \left|\lambda_j\right| \ge \lambda^* > 0 \\
&\qquad 0 < \alpha_{i\,\min} \le \alpha_i \le 1 \qquad (i = 1, \ldots, p)
\end{aligned}
\tag{1}
$$

where C is the structural compliance, **f** and **u** are the global load and displacement vector, respectively. **K** is the global stiffness matrix and Ω is the design domain; V_i is the volume fraction constraint for material phase *i;* λ_j denotes the jth buckling load factor and λ^* is the minimum buckling load factor. α_i denotes design variables of relative density for each different phases at the element level and $\alpha_{i\min}$ is the lower bound of design variables, e.g. $\alpha_{i\min} = 0.001$ to avoid numerical singularity in computation. p is the number of the material. J denotes the buckling mode index set. For the structures which applied loads always stay in the same direction, in the other word, the load direction does not change during the optimization process. The negative buckling load factors are not real, therefore, only the positive buckling load factors should be included in set J.

2.2. Optimization solver

The ideal method for structural optimization should be flexible, general, and able to handle not only element size as design variables, but also other variables such as shape and material orientation angles. It should be able to handle all kinds of constraints. The Method of Moving Asymptotes (MMA) is an effective method for structural optimization that is based on a special type of convex approximation [19]. In current topology optimization problem, two constraints are applied that are volume and buckling load factor constraints. The classical optimizer such as Optimality Criteria (OC) method does not well solve for multiple constraints problem [9]. While MMA can give a fast and accurate result when solving multiple constraints problem [40]. Therefore, MMA is used as a solver for this multi-material topology optimization problem. MMA procedure requires the update of upper and lower moving asymptotes (L_j, U_j) which depend on the design variables α_i in every main loop. However, the original Gauss-Seidel version for active phase algorithm [15] includes an inner loop and results of one target phase in binary active phase are used as an initial value to calculate the next other binary active phases. The Gauss-Seidel iterations update the design variables sequentially for each binary phase in one main loop. Therefore, this causes the conflicts when