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Brief paper

Discrete mode observability of structured switching descriptor linear systems: A graph-theoretic approach ☆

Taha Boukhobza [1], Frédéric Hamelin

Université de Lorraine, CRAN, UMR 7039, Campus Sciences, BP 70239, Vandoeuvre-les-Nancy Cedex, 54506, France

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ABSTRACT

The main result of the paper consists in necessary and sufficient graphical conditions which ensure the generic discrete mode observability of structured switching descriptor systems. The methods used in the previous studies on the observability of switching linear systems on standard form are not applicable to switching descriptor systems. So, we develop a new approach starting from bipartite representations of these systems and then building a new kind of digraph dedicated to the discrete mode observability study. The proposed method assumes only the knowledge of the system's structure and is applicable to a large class of descriptor systems including regular and non-regular systems even if they are square or under-determined. The provided conditions can be implemented by the classical graph-theory algorithms.

1. Introduction

Hybrid systems, combining event-driven and time-driven dynamics, have received growing attention in the control community as they describe a wide range of systems (Johansson & Rantzer, 2007). On the other hand, descriptor systems, which handle systems with both differential and non-differential relations, result from a convenient and natural modelling process (Müller, 2000). Their applications can be found in robotics, electrical networks, biological and economic systems (Müller, 2000). When the model representing the whole or more generally a part of a system is a singular model (for modelling convenience), the functioning system is then represented by a switching descriptor system and in order to check the functioning mode, we have to observe the discrete mode variable of the switching system. Switching descriptor systems are also particularly suited to handle systems (even in standard form) where the dynamics of the continuous part is not entirely known in each discrete mode. Some practical examples where the switching descriptor models are useful and pertinent are provided in Boukas (2008), Clotet, Ferer, and Magret (2009) and De Koning (2003). The paper focuses on the discrete mode observability of switching descriptor systems. The discrete mode observability is relevant to detect some abrupt changes due to faults and which make the system switching to non nominal dynamics or for supervision when the switching between different modes implies control structure modifications. Few works deal with the observability of switching descriptor system, whereas the developed approaches used to study systems in standard form are not directly applicable. Moreover, for the most part, observability studies use algebraic or geometric approaches and so require the exact knowledge of the state space matrices characterizing the systems' model. In many modelling problems or during the conception stage, these matrices have a number of fixed zero entries determined by the physical laws while the remaining entries are not precisely known. In these cases, to study the structural properties, like observability, the idea is that we only keep the zero/non-zero entries in the state space matrices. Many interesting works on these models, called structured models, aim to analyse their properties (Dion, Commault, & Van der Woude, 2003; Murota, 1987; Reinschke, 1988).

The paper is organized as follows: after Section 2, which is devoted to the problem formulation, some definitions related to the graph-theoretic approach are given in Section 3. The main result is provided in Section 4 before a brief conclusion.

2. Problem statement

Consider the following switching descriptor system (SDS)

$$\Sigma : \begin{cases} E_{r(t)}\dot{x}(t) = A_{r(t)}x(t) \\ y(t) = C_{r(t)}x(t) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$ are respectively the state vector and the output (measurement) vector and where $E_{r(t)} \in \mathbb{R}^{m\times n}$, $A_{r(t)}$

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E-mail addresses: taha.boukhobza@univ-lorraine.fr (T. Boukhobza), frederic.hamelin@cran.uhp-nancy.fr (F. Hamelin).

[1] Tel.: +33 3 83684464; fax: +33 3 83684462.

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$\in \mathbb{R}^{m\times n}$ and $C_{r(t)} \in \mathbb{R}^{p\times n}$. In order to guarantee that there exists at least one trajectory $x(t)$ satisfying the relations defining system Σ, $E_{r(t)}x(0_-)$ is assumed to be admissible *i.e.* it does not result in contrary equations in Σ and is such that system Σ is solvable. The exogenous and unobserved discrete mode variable (or switching signal) $r : [0, \infty) \to Q = \{1, \ldots, N\}$, is assumed, as in Babaali and Pappas (2005), to be right-continuous and only a finite number of jumps can occur in any finite interval.

The discrete mode observability is the capacity to deduce the discrete mode knowing the measurements. It is based on the mode distinguishability:

Definition 1 (*Mode Distinguishability*)**.** Two distinct modes $q \in Q$ and $q' \in Q$ are distinguishable if, for almost all initial conditions x_0, either there exist an integer $s \geq 0$ and an expression $f_q(y, \dot{y}, \ldots, y^{(s)}) = 0$ which is satisfied for mode q but is not satisfied for mode q', or there exist an integer $s' \geq 0$ and an expression $f_{q'}(y, \dot{y}, \ldots, y^{(s')}) = 0$ which is satisfied for mode q' but is not satisfied for mode q.

Here, "for almost all initial conditions x_0" is to be understood as "for all $x_0 \in \mathbb{R}^n$ except for the zero set of some polynomials with real coefficients in the n initial state components" ($x_0 = 0$ for example).

Definition 2 (*Discrete Mode Observability*)**.** Σ is discrete mode observable if its modes are distinguishable 2-by-2.

Discrete mode observability analysis can then be reduced to the study of the distinguishability of each pair of modes. Thus, in this paper, we consider that we have only 2 modes. Moreover, since we study a structural property, it is pertinent to deal with structured systems, for which we assume that only the sparsity pattern of matrices E_q, A_q and C_q is known for $q \in \{1, 2\}$. So, to each entry of these matrices, we only know whether its value is fixed to zero, or that it has a non-fixed real value represented by a parameter λ_i. The vector of these parameters is $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_h)^T$ and it is assumed that Λ can take any value in $\mathbb{R}^h$. We denote by A_q^λ, C_q^λ and E_q^λ respectively the matrices obtained by replacing the non-zeros in A_q, C_q and E_q, for $q \in \{1, 2\}$ by the corresponding parameters λ_i and we denote

$$\Sigma_\Lambda : \begin{cases} E_{r(t)}^\lambda \dot{x}(t) = A_{r(t)}^\lambda x(t) \\ y(t) = C_{r(t)}^\lambda x(t). \end{cases} \qquad (2)$$

If all parameters λ_i are numerically fixed, we obtain a so-called admissible realization of Σ_Λ. We say that a property is true generically for Σ_Λ if it is true for almost its realizations or equivalently for almost all parameters λ_i.

For the discrete mode observability analysis, it is pertinent and necessary to highlight the similarities and the differences between the models associated to these modes. Indeed, for $q \neq q'$, it is not realistic to assume that all the parameters of A_q^λ, C_q^λ or E_q^λ are free from the ones of $A_{q'}^\lambda$, $C_{q'}^\lambda$ or $E_{q'}^\lambda$. Thus, we decompose each structured matrix into 2 parts: the first one is common to the 2 modes and the second one is specific to each mode *i.e.* for $q \in \{1, 2\}$, $A_q^\lambda = A_0 + A_q^s$, $C_q^\lambda = C_0 + C_q^s$ and $E_q^\lambda = E_0 + E_q^s$. We assume that the entries of these matrices are free and that a coefficient of A_q^λ (resp. C_q^λ and E_q^λ) is exclusively in A_0 or in A_q^s (resp. in C_0 or in C_q^s, and in E_0 or in E_q^s). These notations can be extended to the multi-mode case (Boukhobza & Hamelin, 2011a).

3. Graphical representation and definitions

For each mode $q = 1, 2$, we associate to structured system Σ_Λ a bipartite graph noted $B(\Sigma_\Lambda, q) = (\mathbf{V}^+, \mathbf{V}^-, W_q)$, where $\mathbf{V}^+$ and $\mathbf{V}^-$ are 2 disjoint vertex subsets and W_q is the edge set related to mode q. The vertices are associated to the whole internal state, dynamical variables and outputs of Σ_Λ and the edges represent links between these variables for each mode. More precisely, $\mathbf{V}^+ = \mathbf{X}$ and $\mathbf{V}^- = \mathbf{Y} \cup \mathbf{Z}$, with $\mathbf{X} = \{\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}\}$, $\mathbf{Z} = \{\mathbf{z_1}, \mathbf{z_2}, \ldots, \mathbf{z_m}\}$ representing relation $z = E_{r(t)}^\lambda x$ and $\mathbf{Y} = \{\mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_p}\}$.

Edge set is related to each mode q and is defined by $W_q = A_q$-*edges* $\cup C_q$-*edges*$\cup E_q$-*edges*, where $A_q\text{-}edges = \{(\mathbf{x_j}, \mathbf{x_i}) \mid A_q^\lambda(i,j) \neq 0\}$, $C_q\text{-}edges = \{(\mathbf{x_j}, \mathbf{y_i}) \mid C_q^\lambda(i,j) \neq 0\}$ and $E_q\text{-}edges = \{(\mathbf{x_j}, \mathbf{z_i}) \mid E_q^\lambda(i,j) \neq 0\}$. Each edge is associated to a free non-zero parameter of the system's model called the weight of the edge. Number q is written under each edge associated to an element of specific matrices A_q^s, C_q^s and E_q^s and represents its index. The edges which are common to the two modes *i.e.* associated to matrices A_0, C_0 and E_0 have index 0. The edges which are specific to mode q have index q.

Example 1. To the system defined by the following matrices, we associate bipartite graphs in Fig. 1.

$$A_0 = \begin{pmatrix} 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 \\ 0 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 \end{pmatrix},$$

$$C_0 = \begin{pmatrix} \lambda_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_8 & 0 & 0 \end{pmatrix},$$

$$E_0 = \begin{pmatrix} \lambda_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_{10} \\ 0 & \lambda_{11} & \lambda_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{13} & 0 & 0 \end{pmatrix}.$$

The specific matrices for mode 1 are such that the entries of A_1^s are zero except $A_1^s(3, 1) = \lambda_{14}$, $C_1^s = 0$ and the entries of E_1^s are zero except $E_1^s(2, 3) = \lambda_{15}$. The specific matrices for mode 2 are such that $A_2^s = 0$, $C_2^s = 0$ and the entries of E_2^s are zero except $E_2^s(3, 1) = \lambda_{16}$.

- Two edges are disjoint if they have no common vertex. A matching is a set M of disjoint edges.
- A path P is denoted $P = \mathbf{v_{s_0}} \to \mathbf{v_{s_1}} \to \cdots \to \mathbf{v_{s_i}}$, where, for a given $q \in \{1, 2\}$, $(\mathbf{v_{s_j}}, \mathbf{v_{s_{j+1}}}) \in W_q$ for $j = 0, 1, \ldots, i-1$. We say in this case that P covers $\mathbf{v_{s_0}}, \mathbf{v_{s_1}}, \ldots, \mathbf{v_{s_i}}$. A path is simple when every vertex occurs only once. The weight of P is the product of the weights of all its edges. A *cycle* is a path of the form $\mathbf{v_{s_0}} \to \cdots \mathbf{v_{s_i}} \to \mathbf{v_{s_0}}$, where $\mathbf{v_{s_0}}, \ldots, \mathbf{v_{s_i}}$ are distinct.
- Let $\mathcal{V}_1$ and $\mathcal{V}_2$ represent two subsets, P is a $\mathcal{V}_1$-*topped* path if its end belongs to $\mathcal{V}_1$.

Consider now any bipartite graph noted B defined by the triplet $(\mathbf{V}^+, \mathbf{V}^-, W)$, and let us recall the subdivision of such graph into $\nu + 2$ partially ordered irreducible components denoted $\mathcal{C}_i(B) = (\mathbf{V_i^+}(B), \mathbf{V_i^-}(B), W_i(B))$ using the Dulmage–Mendelsohn (DM) decomposition (Dulmage & Mendelsohn, 1958; Murota, 1987):

- ↬ find a maximal matching M in B. We associate to this maximal matching a non bipartite digraph noted $B_M = (\mathbf{V}^+, \mathbf{V}^-, W_M)$ where $(\mathbf{v_1}, \mathbf{v_2}) \in W_M \Leftrightarrow (\mathbf{v_1}, \mathbf{v_2}) \in W$ or $(\mathbf{v_2}, \mathbf{v_1}) \in M$. We denote by $\partial^+\mathbf{M}$ (resp. $\partial^-\mathbf{M}$) the set of vertices in $\mathbf{V}^+$ (resp. in $\mathbf{V}^-$) covered by the edges of M. We note $\mathbf{S_0^+} = \mathbf{V}^+ \setminus \partial^+\mathbf{M}$ and $\mathbf{S_0^-} = \mathbf{V}^- \setminus \partial^-\mathbf{M}$.
- ↬ $\mathbf{V_0^+}(B) = \mathbf{S_0^+} \cup \{\mathbf{v} \in \mathbf{V}^+, \exists$ a path in B_M from $\mathbf{S_0^+}$ to $\mathbf{v}\}$.