Advances in Engineering Software 000 (2017) 1–11

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Research paper

Several explanations on the theoretical formula of Helmholtz resonator

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ARTICLE INFO

Article history:
Received 28 June 2017
Revised 19 July 2017
Accepted 6 August 2017
Available online xxx

Keywords:
Helmholtz resonator
Finite element
Acoustic pressure
Mode shape
Impedance

ABSTRACT

Helmholtz resonator is one of the most basic acoustic models in acoustic theoretical research and engineering applications. It is simple and effective to directly apply the theoretical formula for its resonant frequency calculation, but sometimes the calculation error is too large or even wrong. In this paper, the characteristics of Helmholtz resonators are studied based on the finite element numerical analysis. The influence of structural parameters and boundary conditions of the Helmholtz resonator on the resonant frequency is given, several related problems of the theoretical formula are supplemented and the relevant conclusions are obtained. The explanations employed in this paper could be used as a supplement to the theoretical formula for theoretical study and engineering applications of Helmholtz resonators.

1. Introduction

Helmholtz resonator is one of the most basic acoustic resonant structures, which consists a short tube and an acoustic cavity, as shown in Fig. 1. About the Helmholtz resonator, there are many theoretical studies in the field of acoustics, and there are many direct or indirect applications in engineering, such as the muffler of vehicle's exhaust tube, the intubation expansion chamber, and the porous plate sound absorbing material, etc. [1–5].

The theoretical formula of the Helmholtz resonator is simple, but some pertinent questions on the resonant frequency are poorly described in references, which lead to used blindly. For example, perforated plates are often equivalent to multiple Helmholtz resonators in parallel, the theoretical formula can be used to calculate the resonant sound absorption frequency, but significant errors may exist. When the perforated plate is used, it is sometimes necessary to fill the sound absorbing material on wall surfaces to enhance the sound absorption effect. As a result, the boundary condition no longer satisfies the sound hard boundary of the theoretical formula, in this case the use of theoretical formula to calculate the resonant frequency lead to large error.

Based on the acoustic finite element analysis, the Helmholtz resonator is calculated under various parameters and working conditions, some relevant conclusions can be obtained as a supplement to the theoretical formula.

2. Theoretical formula

As shown in Fig. 1, The Helmholtz resonator can be equivalent to a single-freedom vibration system. In the low frequency, the air in short tube is equivalent to mass in vibration system or inductance in electricity; the air in cavity is equivalent to spring in vibration system or capacitance in electricity. The theoretical formula for the Helmholtz resonator is

$$f = \frac{c_0}{2\pi}\sqrt{\frac{S}{V \cdot l_a}} \quad (1)$$

where f is the resonant frequency, C_0 is the velocity of sound in the air, S is the cross-sectional area of the short tube, V is the volume of the acoustic cavity, and l_a is the total length of the short tube, the actual length of the short tube is l. l_a is l plus correction length.

The correction length of the short tube depends on the acoustic radiation at the two ends of the short tube. The effective length of the tube is increased by Δl than the actual length. If it is a separate Helmholtz resonator, the correction length is $0.85a$, a is the radius of short tube; if the Helmholtz resonator is a bypass tube to a pipe, the correction length of tube is $1.7a$.

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http://dx.doi.org/10.1016/j.advengsoft.2017.08.004
0965-9978/

Table 1
Natural frequency of Helmholtz resonator of Example 1.

Acoustic mode	Natural frequency (Hz)	Acoustic mode	Natural frequency (Hz)
1	184.18	14	4045
2	1372.2	15	4188.1
3	2013.4 (2)	16	4290 (2)
5	2424.5 (2)	18	4395.1
7	2704.8	19	4518.1 (2)
8	3336.9 (2)	21	4590.2 (2)
10	3365.7 (2)	23	4784.1 (2)
12	3599 (2)	25	4955.8

Table 2
Cutoff frequency of Helmholtz resonators.

Radius of cylindrical cavity (m)	Theoretical cutoff frequency (Hz)	Numerical cutoff frequency (Hz)	Error
0.04	2513.8	2516.6	0.111%
0.05	2011.0	2013.4	0.119%
0.06	1675.9	1677.8	0.113%
0.07	1436.4	1438.2	0.125%
0.08	1256.9	1258.4	0.119%
0.09	1117.2	1118.6	0.125%
0.1	1005.5	1006.7	0.119%

Table 3
Effect of geometric parameters on resonant frequency.

Radius of cylindrical cavity (m)	Numerical resonant frequency (Hz)	Underside length of block (m)	Numerical resonant frequency (Hz)
0.05	184.34	0.10	185.71
0.06	186.10	0.13	186.48
0.07	186.49	0.16	185.81
0.08	186.37	0.19	183.96
0.09	185.83	0.22	181.06
0.10	184.92	0.25	177.09

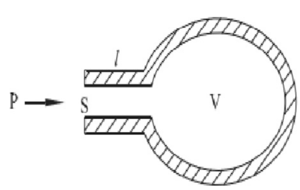

Fig. 1. Schematic diagram of the Helmholtz resonator.

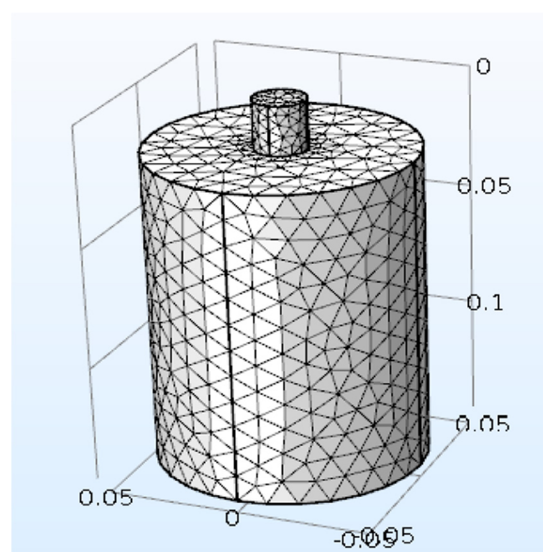

Fig. 2. Acoustic finite element model.

3. Some research on theoretical formula

3.1. Relation between Helmholtz resonant frequency and modal

The resonant frequency of Helmholtz resonator has several orders, but the first order is studied generally. The resonant frequency calculated by theoretical formula corresponds to the first order natural frequency. Under the plane wave excitation, all plane wave modes can be excited by resonance. If using appropriate acoustic excitation, the other modes can also cause resonance, but the application of circumferential and radial modes is less.

Example 1. The Helmholtz resonator with a right cylindrical cavity is selected. The length of the short tube is 0.02 m, the radius of the short tube is 0.01 m, the radius of the cylinder cavity is 0.05 m, the volume of the cylinder cavity is 0.001 m^3.

In this example, the Helmholtz resonator is separate, and the resonant frequency calculated from the theoretical formula Eq. (1) is then 181.25 Hz. The acoustic finite element model is built, as shown in Fig. 2, the analyzing frequency is selected that the maximum size of the mesh is less than 1/6 of the minimum wavelength. The acoustic finite element equation is

$$[K_F + i\omega C_F - \omega^2 M_F]\{p\} = \{F_A\} \tag{2}$$

In the formula, F_A is the acoustic excitation, which is proportional to the normal velocity on the boundary condition. K_F, M_F, C_F are the fluid stiffness, mass and damping matrix respectively, p is