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Research paper

Performance of global metamodeling techniques in solution of structural reliability problems

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ABSTRACT

Solution of structural reliability and uncertainty propagation problems can be a computationally intensive task, since complex mechanical models have to be solved thousands or millions of times. In this context, surrogate models can be employed in order to reduce the computational burden. This article compares the performance of three global surrogate modeling techniques in the solution of structural reliability problems. The paper addresses artificial neural networks, polynomial chaos expansions and Kriging metamodeling. Analytical and numerical problems of increasingly complexity are addressed, including an eight-hundred bar, 3D steel lattice tower. Implementation strategies concerning data mapping and optimization of Kriging hyper parameters are proposed and discussed. Advantages and limitations of each technique are addressed. Results show that the three techniques explored herein are reliable tools for approximating the response of complex mechanical models.

1. Introduction

Solution of structural mechanics problems, nowadays, demands high fidelity, computationally intensive numerical models, such as Finite Element, Boundary Element or finite difference models. Evaluating the reliability of structural systems requires repetitive solution of these numerical models. Approximate solutions have been developed, such as the First Order Reliability Method (FORM) [29]. However, for highly non-linear problems, for problems with multiple failure modes, or for system reliability, FORM is not accurate enough. Monte Carlo Simulation (MCS) can always be employed as an accurate alternative, but it is computationally intense. Evaluating the reliability of highly reliable structures by MCS can easily reach millions of solutions of the high fidelity numerical model. Even for modern computers, the computational burden may become prohibitive. When Reliability-Based Design Optimization (RBDO) is considered, the computational cost skyrockets, because each loop of the optimization algorithm demands a complete reliability analysis [4]. In this context, metamodeling techniques have been developed. Metamodels serve to approximate (or mimic) the response of high-fidelity numerical models, and are much cheaper to evaluate [55]. Once a metamodel is built, the computational cost of the solution becomes virtually irrelevant; hence MCS can be employed for reliability analysis, for instance.

This paper addresses the solution of structural reliability problems by means of metamodeling techniques. The first metamodeling applications to structural reliability involved polynomial response surfaces; in this context, the original methods were called Response Surface Methods [22]. Polynomial response surfaces are local approximations to the high-fidelity model. Polynomial coefficients are obtained by linear regression, based on a set of pre-computed high fidelity responses. Response surface methods have been employed in combination with FORM; however, because they are local approximations, the response surfaces have to reconstructed iteratively, during search for the design points [38]. These original implementations suffered from the same drawbacks of FORM, that is: lack of accuracy for highly non-linear problems, or for multiple failure modes (system reliability). Also because of their local approximation characteristic, polynomial response surfaces are not accurate enough for solutions via Monte Carlo simulation.

In this paper, more recent, global surrogate modeling techniques are addressed. Global models are able to approximate the high fidelity model over the entire domain of the random variable space. When employed in conjunction with MCS, they can provide accurate solutions to non-linear or to system reliability problems. Three global metamodeling techniques are investigated herein, with respect to their accuracy and efficiency in solving structural reliability problems: Artificial Neural Networks (ANN), Polynomial Chaos Expansions (PCE) and Kriging.

Hornik et al. [31] proved that sufficiently complex multilayer feed-forward neural networks are capable of approximating any

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measurable function arbitrarily well. Papadrakakis et al. [48] applied ANN in Monte Carlo simulation for reliability analysis for the first time. This successful approach has been improved in many relevant works that followed. Hurtado and Alvarez [33], compared back propagation multi layer perceptron and radial basis function networks in the context of structural reliability. Gomes and Awruch [27] compared the usage of polynomial response surfaces with ANN surrogates, and obtained promising results. Artificial Neural Networks have been employed in several applications, as follows. Zhang and Foschi [60] employed ANN in seismic reliability analysis; Papadrakakis et al. [49] employed ANNs for reliability-based design optimization of large structural systems. Further academic examples were addressed by Cardoso et al. [12]. Firouzi and Rahai [23] performed reliability based inspection of concrete bridge decks considering extent of corrosion-induced cracks using ANN, concluding that the high computational costs involved are efficiently reduced by this approach. Gomes and Beck [28] studied global structural optimization under uncertainties applying ANN to surrogate objective functions, solving the problem with greatly reduced computational effort. A broad literature review about ANN applications to structural reliability can be found in Chojaczyk et al. [13].

The Kriging technique dates back tom the 1950s, when the South African mining engineering Daniel Krige observed how to better predict the average grade in a prospective mining block. He suggested that the sample mean of nearby core-sample assays was not a good predictor, and that the information was somewhat spatially correlated. A broad review of early works about Kriging can be found in Cressie [14]. Kriging surrogate models were first applied to structural reliability problems by Romero et al. [51]. Kaymaz [37] compared the performance of Kriging surrogates with polynomial response surfaces. Echard et al. [19] proposed an adaptive learning method to update Kriging surrogate design of experiments with novel information gathered during the problem's resolution, and coupled it with crude Monte Carlo simulation. Dubourg et al. [16] proposed the solution of RBDO problems using Kriging surrogates to simulate performance functions. Echard et al. [21] coupled Kriging surrogates with importance sampling Monte Carlo Simulation and showed that this approach is efficient even for problems involving small failure probabilities and complex numerical models. Jia and Taflanidis [35] applied Kriging to predict the behavior of hurricanes, and compared it with moving least squares response surfaces. Dubourg and Sudret [17] used the Kriging surrogate to devise a quasi-optimal instrumental density function for computing failure probability through importance sampling. Gaspar et al. [25] presented a thorough assessment of the efficiency of Kriging surrogates in the context of structural reliability.

Polynomial chaos expansions were introduced in structural analysis by Ghanem and Spanos [26], in what was called the Stochastic Finite Element Method (SFEM). Many important applications followed. Anders and Hori [1,2]proposed a SFEM for nonlinear elasto-plastic bodies. Ngah and Young [44] used SFEM to predict stress and strain fields of composite structures with variable material constitutive properties. Sachdeva et al. [52] studied the efficiency of PCE approach on the settlement of a foundation supported on random heterogeneous soil. Several recent studies have addressed problems of structural reliability with PCE surrogating limit states, such as the works of Sudret et al. [56] and Riahi et al. [50], among others. Blatman and Sudret [10] derived an algorithm to reduce PCE limitations in problems with many random variables, building sparse basis for the expansions, ignoring terms of minor relevance. Spiridonakos and Chatzi [53] studied nonlinear structural dynamic systems using PCE to nonlinear autoregressive exogenous models (NARX), resulting in what was called PCE-NARX models.

Even though several works have investigated the usage of surrogate models in different applications, not many have been dedicated to compare the performances of different surrogate models. ANN metamodels had its performances compared to polynomial response surfaces in the work of Gomes and Awruch [27]. Kriging surrogates were compared to polynomial response surfaces by Kaymaz [37]. Gano et al. [24] compared Kriging with second order regressions and with a commercial application called Datascape. Kriging was also compared to radial basis functions and multivariate adaptive regression splines metamodels for the specific problem of water injection optimization in the work of Babaei and Pan [5]. However, there is no published work directly comparing the performance of the PCE, ANN and Kriging in the context of structural reliability. These three metamodels are investigated herein, and employed in solution of analytical and numerical problems of increasing complexity. Implementation details are proposed and discussed. Advantages and limitations of each technique are addressed.

The remainder of this paper is organized as follows. The problem statement is presented in Section 2. The three surrogate modeling techniques are described in Section 3. The application examples are presented in Section 4, and concluding remarks close the paper in Section 5.

2. Problem statement

Let $\boldsymbol{X}$ be a vector that gathers together the m random input parameters of the model (e.g. geometrical and material properties, loads, etc.) with prescribed density function $f_{\boldsymbol{X}}(\boldsymbol{x})$. The uncertainty implies in the possibility of undesirable structural responses, mathematically described by a limit state function $g(\boldsymbol{x})$, such that:

$$\Omega_f = \{\boldsymbol{x} | g(\boldsymbol{x}) \leq 0\} \quad \text{is the failure domain} \tag{1}$$

$$\Omega_s = \{\boldsymbol{x} | g(\boldsymbol{x}) \rangle 0\} \quad \text{is the survival domain} \tag{2}$$

The probability of failure of the system is defined as:

$$P_f = \int_{\Omega_f} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{3}$$

Solution of Eq. (3) cannot be obtained in closed form, because the integration domain is implicitly defined. In the FORM method, Eq. (3) is solved by a mapping to standard Gaussian space, and by a linearization of the limit state function (integration domain) at the so-called design point.

A more robust and accurate, yet computationally expensive solution is obtained by Monte Carlo Simulation (MCS). In this technique, the failure probability is interpreted as the mean value of a stochastic experiment where a large number of random variable samples are generated [15]. An indicator function $I(\boldsymbol{x})$ is used, which assumes value 0 over the survival domain and 1 over de failure domain, and n samples of $\boldsymbol{X}$ are generated following $f_{\boldsymbol{X}}(\boldsymbol{x})$. The probability of failure is then estimated as the sample average:

$$\hat{P}_f = \frac{1}{n} \sum_{i=1}^{n} I(\boldsymbol{x}_i) \tag{4}$$

It is well known that $\hat{P}_f$ is an unbiased estimator of P_f, i.e., in the limit with $n \to \infty$, $\hat{P}_f \to P_f$. In practice, large number of samples n is used, in order to reduce the variance of the estimator. Hence, n could be of the order of millions. In order to avoid millions of runs of the high fidelity model, metamodels can be constructed at reduced computational cost.