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Brief paper

Analysis and application of a novel fast algorithm for 2-D ARMA model parameter estimation ☆

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ABSTRACT

In this paper, we analyze a novel algorithm for 2-D ARMA model parameter estimation in the presence of noise and then develop a fast and efficient blind image restoration algorithm. We show that the novel algorithm can minimize a quadratic convex optimization problem and has a lower computational complexity than the conventional algorithms. As a result, the novel algorithm involves no convergence and local minimum issue. Moreover, the proposed blind image restoration algorithm can overcome the local minimization problem. Computed results confirm that the novel algorithm can more quickly obtain more accurate estimates than the conventional algorithms in the presence of noise.

1. Introduction

Many engineering problems in image restoration (Kaufman & Tekalp, 1991), texture analysis (Hall & Giannakis, 1995), image encoding (Chung & Kanefsky, 1992), system identification and modeling (Tekalp, Kaufman, & Woods, 1986) can be converted into the parameter estimation problem of 2-D autoregressive moving average (ARMA) processes. To implement this technique, it is necessary to estimate parameters of the 2-D ARMA model. So, how to obtain a good estimate of the 2-D ARMA model parameters is an important topic. Consider the following quarter-plane 2-D ARMA model of order (p_1, p_2, q_1, q_2):

$$\sum_{i=0}^{p_1}\sum_{j=0}^{p_2} a_{i,j}x(n_1-i, n_2-j) = \sum_{m=0}^{q_1}\sum_{n=0}^{q_2} d_{m,n}w(n_1-m, n_2-n) \quad (1)$$

where $a_{0,0} = 1$, $x(n_1, n_2)$ is a stable random field, and $w(n_1, n_2)$ is an unknown random excitation which is assumed to be the white Gaussian noise with zero mean and variance σ_w^2. In practice, the random field $x(n_1, n_2)$ is observed in the presence of noise as

$$y(n_1, n_2) = x(n_1, n_2) + u(n_1, n_2) \quad (2)$$

where $u(n_1, n_2)$ is the measurement noise of zero mean and is assumed to be uncorrelated with $x(n_1, n_2)$, and $1 \le n_1 \le N_1$, $1 \le n_2 \le N_2$ with sample sizes N_1 and N_2. Our objective is to estimate the AR parameters $a_{i,j}(0 \le i \le p_1, 0 \le j \le p_2, (i,j) \ne (0,0))$ and the MA parameters $d_{m,n}(0 \le m \le q_1, 0 \le n \le q_2)$, based on the noisy observations of the random field and the prior knowledge of the model order. As for model order determination techniques, the interested reader may refer to reference papers (Abo-Hammour, Alsmadib, Al-Smadic, Zaqoutc, & M S, 2012; Giannakis & Mendel, 1990).

It is well known that ARMA model parameter estimation is much more difficult than pure autoregressive (AR) or moving-average (MA) model parameter estimation because of the complex intrinsic dependency between the ARMA parameters and the excitation noise. Although the methods for 1-D ARMA and AR model parameter estimation have been well developed (Alimorad & Mahmood, 2010; Chakhchoukh, 2010; Ding, Ding, Liu, & Liu, 2011b; Ding, Liu, & Liu, 2011a, 2010; Ding, Qiu, & Chen, 2009; Ding, Shi, & Chen, 2006; Hong, Soderstrom, & Zheng, 2007; Song & Chen, 2008), those techniques cannot be easily extended to the 2-D ARMA parameter estimation. There exist two conventional algorithms for solving the 2-D ARMA parameter estimation problem. One algorithm was developed by Zhang and Cheng (1991). First, the following 2-D modified Yule–Walker (MYW) equation is solved for AR

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parameters $\{a_{i,j}\}$:

$$\sum_{i=0}^{p_1}\sum_{j=0}^{p_2} a_{i,j} r_{xx}(q_1+l-i, q_2+m-j) = d_{0,0} d_{q_1,q_2} \sigma_w^2 \delta(l,m) \quad (3)$$

where $\delta(l,m)$ is the Kronecker delta function, $d_{0,0}$ and d_{p_1,p_2} are the MA parameters, and $r_{xx}(k_1,k_2)$ denotes the covariance of the random field. Next, the MA parameters $\{d_{i,j}\}$ are obtained by solving a system of nonlinear equations. There is thus a requirement on the convergence condition to solve this system of nonlinear equations. Another algorithm was proposed by Kizilkaya and Kayran (2005). This algorithm is based on an equivalent 2-D AR (EAR) model technique and is a computationally efficient algorithm with an additional assumption that $d_{0,0} = 1$. Unlike the Zhang-Cheng algorithm, the equivalent AR parameters $b_{i,j}$ are first solved by the following 2-D MYW equation

$$\sum_{s=0}^{L_1}\sum_{t=0}^{L_2} b_{s,t} r_{xx}(l-s, m-t) = \sigma_w^2 \delta(l,m). \quad (4)$$

Then the MA parameters are obtained by minimizing the following cost function

$$\Phi_1 = \left(\sum_{k=0}^{q_1}\sum_{j=1}^{q_2} \boldsymbol{B}_{k,j} d_{k,j} + \sum_{h=1}^{q_1} \boldsymbol{B}_{h,0} d_{h,0} + \boldsymbol{B}_{0,0} - \boldsymbol{A}\right)^2 \quad (5)$$

where $\boldsymbol{A}$ and $\boldsymbol{B}_{k,j}$ are matrices with dimension $(L_1+1)\times(L_2+1)$. The AR parameters are finally computed by

$$a_{m,n} = b_{m,n} + \sum_{k=0}^{q_1}\sum_{j=1}^{q_2} B_{k,j}(m,n) d_{k,j} + \sum_{h=1}^{q_1} B_{h,0}(m,n) d_{h,0}. \quad (6)$$

The Kizilkaya–Kayran algorithm does not need solving a system of nonlinear equations. Yet, it involves a high computational cost to solve (5). In addition, the two conventional algorithms do not consider the case of observation noise. Recently, to overcome their disadvantages we introduced a novel algorithm for 2-D ARMA parameter estimation in a conference paper (Deng & Xia, 2009). In this paper, we analyze convergence and computational complexity of the novel algorithm. Furthermore, we develop a fast and efficient blind image restoration method. The proposed blind image restoration method can overcome the local minimization problem.

2. Novel estimation algorithm and analysis

2.1. Novel estimation method

By extending the 2-D EAR inverse filter technique (Kizilkaya & Kayran, 2005), we have the following approximation of random excitation

$$w(n_1,n_2) \approx \sum_{s=0}^{L_1}\sum_{t=0}^{L_2} b_{s,t} y(n_1-s, n_2-t). \quad (7)$$

Substituting (2) and (7) into (1) yields

$$\begin{aligned} y(n_1,n_2) &= -\sum_{i=0}^{p_1}\sum_{\substack{j=0\\(i,j)\neq(0,0)}}^{p_2} a_{i,j} y(n_1-i, n_2-j) \\ &\quad + \sum_{m=0}^{q_1}\sum_{n=0}^{q_2}\sum_{s=0}^{L_1}\sum_{t=0}^{L_2} d_{m,n} b_{s,t} \\ &\quad \times y(n_1-m-s, n_2-n-t) + \bar{n}(n_1,n_2) \end{aligned}$$

where $\bar{n}(n_1,n_2)$ is the additive zero-mean noise given by

$$\bar{n}(n_1,n_2) \approx -\sum_{i=0}^{p_1}\sum_{j=0}^{p_2} a_{i,j} u(n_1-i, n_2-j). \quad (8)$$

Then the 2-D ARMA model parameter estimation is converted into the 2-D AR model parameter estimation. Thus the novel 2-D ARMA parameter estimation method is to solve an optimization problem:

$$\text{Minimize } \Phi_2(\theta) \quad (9)$$

where

$$\begin{aligned} \Phi_2(\theta) &= \frac{1}{2}\sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\Bigg[y(n_1,n_2) + \sum_{i=0}^{p_1}\sum_{\substack{j=0\\(i,j)\neq(0,0)}}^{p_2} a_{i,j} y(n_1-i, n_2-j) \\ &\quad - \sum_{m=0}^{q_1}\sum_{n=0}^{q_2}\sum_{s=0}^{L_1}\sum_{t=0}^{L_2} d_{m,n} b_{s,t} y(n_1-m-s, n_2-n-t)\Bigg]^2 \end{aligned}$$

and $\theta \in R^{(p_1+1)(p_2+1)+(q_1+1)(q_2+1)}$ with elements a_{ij} and $d_{m,n}$.

2.2. Convex analysis of the estimation approach

Now, we show that the novel algorithm can optimize a quadratic convex optimization problem (9).

Theorem 1. *$\Phi_2(\theta)$ is a quadratic convex function.*

Proof. First, it is easy to see that $\Phi_2(\theta)$ is a quadratic function. Next, let

$$\hat{w}(n_1-m, n_2-n) = \sum_{s=0}^{L_1}\sum_{t=0}^{L_2} b_{s,t} y(n_1-m-s, n_2-n-t).$$

Then

$$\begin{aligned} \Phi_2(\theta) &= \frac{1}{2}\sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\Bigg[y(n_1,n_2) + \sum_{i=0}^{p_1}\sum_{\substack{j=0\\(i,j)\neq(0,0)}}^{p_2} a_{i,j} y(n_1-i, n_2-j) \\ &\quad - \sum_{m=0}^{q_1}\sum_{n=0}^{q_2} d_{m,n} \hat{w}(n_1-m, n_2-n)\Bigg]^2. \end{aligned} \quad (10)$$

We have

$$\frac{\partial \Phi_2(\theta)}{\partial a_{s_1,t_1}} = \sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2} y(n_1-s_1, n_2-t_1) e_{n_1,n_2}(\theta)$$

and

$$\frac{\partial \Phi_2(\theta)}{\partial d_{k_1,l_1}} = -\sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2} \hat{w}(n_1-k_1, n_2-l_1) e_{n_1,n_2}(\theta)$$

where

$$\begin{aligned} e_{n_1,n_2}(\theta) &= \Bigg[y(n_1,n_2) + \sum_{i=0}^{p_1}\sum_{\substack{j=0\\(i,j)\neq(0,0)}}^{p_2} a_{i,j} y(n_1-i, n_2-j) \\ &\quad - \sum_{m=0}^{q_1}\sum_{n=0}^{q_2} d_{m,n} \hat{w}(n_1-m, n_2-n)\Bigg]. \end{aligned}$$

Furthermore, we have

$$\frac{\partial^2 \Phi_2(\theta)}{\partial a_{s_1,t_1}\partial a_{s_2,t_2}} = \sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2} y(n_1-s_1, n_2-t_1) \times y(n_1-s_2, n_2-t_2)$$