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Research paper

Robust topology optimization for structures under interval uncertainty

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ABSTRACT

This paper proposes a new non-probabilistic robust topology optimization approach for structures under interval uncertainty, as a complementarity of the probabilistic robust topology optimization methods. Firstly, to avoid the nested double-loop optimization procedure that is time consuming in computations, the interval arithmetic is introduced to estimate the bounds of the interval objective function and formulate the design problem under the worst scenario. Secondly, a type of non-intrusive method using the Chebyshev interval inclusion function is established to implement the interval arithmetic. Finally, a new sensitivity analysis method is developed to evaluate the design sensitivities for objective functions like structural mean compliance with respect to interval uncertainty. It can overcome the difficulty due to non-differentiability of intervals and enable the direct application of gradient-based optimization algorithms, e.g. the Method of Moving Asymptotes (MMA), to the interval uncertain topology optimization problems. Several examples are used to demonstrate the effectiveness of the proposed method.

1. Introduction

In the field of structural optimization, topology optimization has experienced considerable development over the past two decades with a range of applications [1]. Topology optimization is essentially a numerical process to optimize a prescribed objective function under specific constraints by iteratively distributing a given amount of material, until the best layout of the material is obtained in the design domain. Several typical methods have been developed for topology optimization of structures, such as the homogenization method [2], the SIMP based methods [3, 4], and the level set-based methods (LSMs), e.g. [5–7], as well as the heuristic methods like the evolutionary structural optimization (ESO) method and its variants [8, 9].

However, the majority of current studies about the topology optimization of structures are based on the deterministic assumption, which may result in a design that cannot satisfy the expected design goal and even a design that is unfeasible, as most problems in engineering inevitably involve various uncertainties, including the manufacturing tolerance, load variations, inhomogeneity of material properties, and so on [10]. For a structure, the topological design may be quite different when uncertain factors are considered. As a result, the performance of a structure, such as robustness and reliability, is unavoidably subject to variations in practice due to various uncertainties [11,12]. Hence, it is necessary to incorporate uncertainties into structural topology optimization problems quantitatively, in order to enhance structural safety and avoid failure in extreme working conditions.

The reliability-based design optimization (RBDO) [11,13–15] and robust design optimization (RDO) [16–19] are two main methods, which have been used to account for different uncertainties in engineering optimization. RBDO focuses on a risk-based solution taking into account the feasibility of target at expected probabilistic levels, in which the risk is commonly measured by the probabilities of failure. Thus, RBDO seeks a design that achieves a targeted probability of failure (i.e., less than some acceptable and invariably small value) and therefore ensures that the conditions that may lead to catastrophe are unlikely. The RBDO has been combined with topology optimization to deliver the so-called reliability-based topology optimization (RBTO) methods. For instance, Kharmanda et al. [20] studied topology optimization of continuum structures considering uncertainties by using the first-order reliability method. In [21], a non-deterministic topology optimization methodology is proposed by using a hybrid cellular automate method combined with a decoupled RBDO approach. Luo et al. [22] proposed a RBTO method based on a multi-ellipsoid convex model for problems consisting of non-probability uncertainties [23,24], and so on [25].

The RDO aims to reduce the sensitivity of the objective function with respect to uncertain parameters, so it can minimize both the mean and variation of the objective function. The application of

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RDO to structural topology optimization refers to the robust topology optimization, termed as RTO which is the major focus of this research. There have been some studies investigating RTO under uncertainties of load conditions, material properties, and geometry [26]. For instance, Sigmund [27] presented a topology optimization method to include uncertainties during the fabrication of micro and nanostructures [28]. Guest et al. [29] studied a perturbation-based topology optimization method for solving problems with small uncertainty level of externally applied loads. The perturbation method [30] was also used to solve RTO problems with small uncertainty of geometry. Asadpoure et al. [10] combined deterministic topology optimization techniques with a perturbation method for quantification of uncertainties associated with structural stiffness. The main concept of the perturbation method is to transform the original topology optimization problem under uncertainty into an augmented deterministic problem. However, the perturbation method may produce errors which cannot be ignored when the uncertainty level of parameters is relatively high.

For the continuous problems with uncertainty, the stochastic spectrum-based method is usually used to discretize the random field. Tootkaboni et al. [31] combined the polynomial chaos expansion with topology optimization, to design continuum structures to achieve robustness in presence of random uncertainties. Zhao et al. [32] considered loading uncertainty of random field by using the Karhunen–Loeve expansion to characterize the random field as a reduced set of random variables. The Karhunen–Loeve expansion was also used to develop robust topology optimization method [33] with random field uncertainty, in which the univariate dimension-reduction method was combined with the Gauss type quadrature sampling to calculate statistical moments of the objective function. Jansen et al. [34] discretised the random field by using the expansion optimal linear estimation method, which particularly suits for discretising random fields with a relatively large correlation length. Zhao et al. [35] proposed an efficient approach by completely separating the Monte Carlo sampling with topology optimization to solve the RTO problem of structures under loading uncertainty, which obtained the accurate calculation of the objective function.

Most of the aforementioned RTO methods are based on the theory of random field or random variables, using a combination of the first and second order statistical moments (mean and variance) of the design response as the objective function of the RTO problems. However, in engineering, how to accurately describe probability distribution functions is a challenging task, especially for variables with limited uncertainty information. In some cases, for the uncertain variables the lower and upper bounds can be more easily obtained than the evaluation of accurate probability distributions [36]. Hence, the uncertain-but-bounded parameters may be more suitable for describing uncertainties under some situations. When non-probabilistic parameters are used to describe the uncertain parameters, the performance under the worst condition can be used to define the objective function of RTO problems. In [37], the RTO problem was formulated to minimize the maximum compliance induced by the worst case of an uncertain load set, which was characterized by a convex model. By constraining the Euclidean norm of the uncertain loads, the robust optimization problem was formulated as the minimization of maximum eigenvalue of an aggregated symmetric matrix, according to the Rayleigh-Ritz theorem for symmetric matrices. However, this method can only be used to handle the convex model rather than the interval uncertainty. Csébfalvi et al. [38] considered the direction of load as uncertain-but-bounded parameters to optimize the truss by using a nonlinear optimization solver the previously developed hybrid metaheuristic ANGEL [39], but it was not used in the optimization of continuous structure. Wang et al. [36] presented a hybrid genetic algorithm, which was integrated with a simple local search strategy as the worst-case-scenario of an anti-optimization, to tackle structure topology optimization under interval uncertainty. However, the anti-optimization method is usually time-consuming, especially for the RTO of continuum structures, which often involves a nested double-loop optimization process that is computationally expensive.

There have been some applications about the interval uncertainty analysis and optimization. Jiang et al. [40] proposed an optimization method for uncertain structures based on convex model and a satisfaction degree of interval, in which the interval analysis method was used to determine the bounds of constraints. This method was then applied to [41], but the neural network was employed to calculate the bounds of constraints. Gao et al. [42] studied the interval dynamic response of vehicle-bridge interaction systems, in which the parameters of the bridge and vehicle were considered as interval variables, and a heuristic optimization method (LHNPSO) was used to find the bounds of bridge displacement. The hybrid uncertainty analysis of probability and interval uncertainty was also studied in references [43, 44].

Compared to optimization algorithms, the interval arithmetic [45] is a more efficient method that can be applied to handle the interval uncertainty, but it often produces large overestimation [46,47]. A series of techniques have been developed to control overestimation or wrapping effect induced by interval arithmetic, e.g. the Taylor series-based method [48,49], Taylor model method [50,51], and Chebyshev interval method [46,47]. Due to its non-intrusive characteristic, the Chebyshev interval method can be implemented for complex models as a black-box model. The Chebyshev interval method has been applied to the optimization problem of vehicle dynamics for hardpoints coordinate with interval uncertainty [52] and truss structures for geometric dimensions with interval uncertainty [53], and demonstrated as an effective method to compress the overestimation and avoid the nested double-loop in the optimization. However, there has been no publication in applying interval arithmetic to the RTO problems of continuum structures, particularly due to the following numerical issue.

Besides the overestimation in the numerical implementation, there has been no effective method so far developed for commutating the derivatives of interval functions, due to non-differentiability of intervals. However, the first-order derivatives of the objective function with respect to the design variables are often required to enable the application of the gradient-based mathematical programming methods to the RTO problems. Therefore, the difficulty for computing the derivatives is another important issue in applying the interval arithmetic to the RTO problems. In this paper, the Chebyshev interval method [46,47] will be introduced to the RTO problems of continuum structures with uncertain-but-bounded parameters. The interval functions of RTO problems will be calculated by the interval arithmetic, in order to improve computational efficiency by avoiding the nested double-loop optimization and numerical accuracy by compressing the overestimation due to interval wrapping effect. In particular, a new numerical scheme will be developed to compute the derivatives of interval functions, which makes it possible to implement the RTO problems by using many traditional but efficient gradient-based optimization algorithms.

2. Material density based approach for topology optimization

A typical topology optimization problem is the one to find the best layout of material within a given design domain, to minimize a prescribed objective function while satisfying a set of constraints. The well-known topological optimization design problem is the minimization of structural mean compliance. With a given amount of material, the goal of the optimization is to identify the