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Response analysis of longitudinal vibration of sucker rod string considering rod buckling

Mingming Xing

College of Mechanical Engineering, Linyi University, Linyi City, Shandong Province, 276000, China

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ABSTRACT

Considering the limitation of simulation model of the deep well and ultra-deep well, an improved simulation model of sucker rod pumping system (SRPS) is presented. The main innovations of the simulation model are nonlinear longitudinal vibration model of sucker rod string (SRS) and numerical integral model of nonlinear system with pump time varying parameters. In the detail, the relationship of rod buckling and plunger load is analyzed using a finite element preprocessor module of ANSYS. Based on analysis results and experimental results, the change rule of equivalent stiffness is studied. Using parameters of oil well, the response of longitudinal vibration of SRS is analyzed. The new model is verified by comparing it with the available analytical solution, and good agreement is found. The simulation results show that the instantaneous equivalent stiffness of helical buckling section of SRS follows exponential distribution. The equivalent stiffness is affected by some parameters, among them, the values of water cut, density of crude oil and pump depth have a significant effect. With new model presented in this paper, the subharmonic resonance is found, which is characteristic of nonlinear vibration. Comparing with the steel rod, carbon fiber rod and fiberglass rod, the resonant phenomenon of single rod is more likely to happen for the wire rope sucker rod. However, when sucker rod is double stage rod, the resonant phenomenon of fiberglass rod is more likely to happen than other rods. The above results are significant to the optimization of SRS.

1. Introduction

The SRPS is made up of three parts. The first part is surface driving unit, which converts rotary motor to linear motion. The second part is pump that is installed on the lower end of tube string, which absorbs and expels the liquid in oil well. The last part is sucker rod string, which transmits the movement and force of surface driving unit to the pump. The three-dimensional map of SRPS is shown in Fig. 1. The sucker rod string is a very slender and big flexible rod, which is affected by many nonlinear factors. As the steady response of SRPS is greatly influenced by the longitudinal vibration of SRS, the response analyses of SRS's longitudinal vibration is increasingly attracting the attention of experts and researchers in the word [1–3]. The most successful simulation model describing the sucker rod dynamics is developed by Gibbs [4]. Since then, the simulation model of SRS's longitudinal vibration is increasingly applied based on the Gibbs's wave equation. Tripp, Jennings and Gibbs et al put forward the application of fiberglass sucker rods based on the Gibbs's wave equation [5–7]. Everitt presented a finite difference representation of the wave equation developed for diagnostic analysis of SRPS [8]. With wave equation, an improved model of longitudinal vibration of liquid columns and SRS is built by Doty, Schmidt and Lekia [9,10]. Recently, the boundary conditions that affect rod law of motion are analyzed and the excitation forces of SRS's longitudinal vibration are corrected by DaCunha [11].

The rods below the neutral point of SRS will lack of stability and become deformed spirally with the action of load on the down-stroke. According to domestic and international statistic data, almost 70 percent breaking accidents of SRS is breakdown below the neutral point of SRS, which has relation with deformed spirally of SRS [12]. Therefore, characteristic analysis of SRS considering rod buckling is significant in the design and use of SRS. With Lubinski's study results of drilling string [13], the formula of critical instability load of rod buckling is derived by Long S.W, Li et al with energy method [14,15]. Nickens studied the effect of pump clearances on possible rod buckling above the pump [16]. Mitchell, Robert and Halliburton presented two concentric pipes can interact when one or both pipes are in compression and would then have a tendency to buckle [17].

In all aforementioned approaches, a lot of simulation models are built and employed in oilfield, but they have not been studied sufficiently. The effect of longitudinal vibration of SRS on rod load is studied by some experts [1,4,5], and the effect of rod buckling

E-mail address: xingmingming2009@126.com

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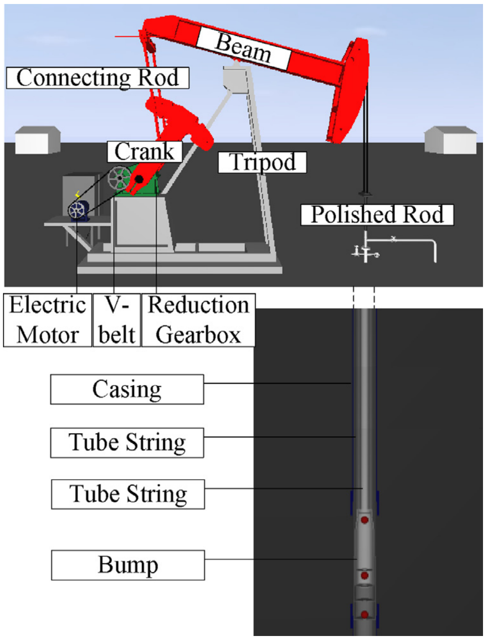

Fig. 1. Sucker rod pumping system.

on rod load is studied by some experts [13,14,17]. However, what is the relationship between the longitudinal vibration of SRS and rod buckling, and whether the longitudinal vibration of SRS is affected by rod buckling. Besides, the tension rigidity of SRS is constant or variable with rod buckling, where the response of longitudinal vibration of SRS considering rod buckling is ignored. Focusing on these problems, there is no relevant conclusion currently. Therefore, the studies of following respects have been done.

In this paper, in Section 2, an improved simulation model of SRPS is derived based on the time-varying stiffness of rod buckling. In the detail, the transmission device of SRPS is simplified to equivalent crank of a single-degree-freedom system, and the SRS is divided into a spring-mass-damper system, considering the effect of instantaneous equivalent stiffness. In Section 3, the analytical solution of SRPS is given, which solves the problem that the dynamic responses of SRPS cannot be achieved directly. In the detail, instead of the finite difference method, the numerical integral method is used to solve steady-state response of rod-tube-pump system. In Section 4, the relationship of rod buckling and plunger load is analyzed using a finite element preprocessor module of ANSYS. Based on analysis results, the change rule of equivalent stiffness is given. Then the affecting factors of equivalent stiffness of helical buckling section are analyzed. Considering the resonant phenomenon of SRS, the design of rod string assemblage is also analyzed. In Section 5 the signification of dynamic simulation research on longitudinal vibration of SRS and the conclusions are summarized.

2. Simulation model of SRPS

In order to establish simulation model of SRPS, the fundamental assumptions are as follows.

(1) The effect of motor speed fluctuation on the boundary conditions of SRS's vibration is considered.
(2) The viscous damping of liquid is concerned.
(3) The oil well is vertical.
(4) The direction from bottom-up is described as positive.
(5) The lateral vibration of the SRS can be neglected.
(6) The effect of rod buckling on SRS's longitudinal vibration is considered.

Based on above assumptions, a mechanical model of dynamic simulation of SRPS is established, as follows

In Fig. 2 (a), the surface driving system of beam pump system is simplified as a single degree of freedom system. With beam pumping unit in Fig. 2 (b), the crank angle is changed into the polish rod displacement that is excitation of SRS's longitudinal vibration. Fig. 2 (c) shows that SRS is a spring-mass-damper system with constant stiffness in $0\sim L_{\rm np}$ units and with non-constant stiffness in $L_{np}\sim L_n$ units, and the neutral point of SRS is a cutoff point. The equivalent stiffness and damping of shock absorber of SRS is k_{e0}, c_{e0}. The displacement, velocity and acceleration for polished rod are defined with u_0, $\dot{u}_0$, $\ddot{u}_0$. Then a mathematical model of dynamic simulation of SRPS is established, as follows.

$$\begin{cases} J_c\ddot{\theta} + \dfrac{1}{2}\dot{\theta}^2\dfrac{dJ_c}{d\theta} = M_d - M_f \\ u_0(t) = \left\{ \arccos\left(\dfrac{L_C{}^2 + L^2 - L_{Pb}{}^2}{2L_C L}\right) + \arcsin\left[\dfrac{r_c}{L}\sin\left(2\pi - \right.\right.\right. \\ \left.\left.\left. \theta_0 + \theta + \arcsin\dfrac{L_I}{L_K}\right]\right) - \arccos\left(\dfrac{L_C{}^2 + L_K{}^2 - (L_{Pb} - r_c)^2}{2L_C L_K}\right)\right\} L_A \\ m_1(\ddot{u}_1 + \ddot{u}_0) + k_{e0}u_1 + c_{e0}\dot{u}_1 + k_1(u_1 - u_2) + c_1(\dot{u}_1 + \dot{u}_0) = 0 \\ \vdots \qquad \cdots \qquad \vdots \qquad \vdots \\ m_i(\ddot{u}_i + \ddot{u}_0) + k_{i-1}(u_i - u_{i-1}) + k_i(u_i - u_{i+1}) + c_i(\dot{u}_i + \dot{u}_0) = 0 \\ \vdots \qquad \cdots \qquad \vdots \qquad \vdots \\ m_n(\ddot{u}_n + \ddot{u}_0) + k_n(u_n - u_{n-1}) + c_n(\dot{u}_n + \dot{u}_0) - P_p(t) = 0 \end{cases} \tag{1}$$

Where

$$\begin{cases} m_i = \dfrac{\rho_r l_{0i}\pi D_i{}^2}{4} i = 1, 2, 3\cdots n \\ k_x = \begin{cases} \dfrac{E_r A_{ri}}{l_{0i}} & 0 < x \le L_{np} \\ H(-\dot{u}_0)\dfrac{E_r A_{ri}k_{ig}(t)}{E_r A_{ri} + k_{ig}(t)l_{0i}} + H(\dot{u}_0)\dfrac{E_r A_{ri}}{l_{0i}} & L_{np} < x \le L_n \end{cases} \\ c_i = \dfrac{12\pi\mu L_r}{n}\left(\dfrac{D_i}{D_{ti} - D_i}\right)\left[\left(0.20 + 0.39\dfrac{D_i}{D_{ti}}\right)\right. \\ \qquad \left. + \dfrac{2.197\times 10^4}{25}\left(\dfrac{D_{ci}}{D_{ti}} - 0.381\right)^{2.57}\dfrac{D_{ci}{}^2 - D_r{}^2}{l_0 i D_i}\right] \end{cases} \tag{2}$$

Where, the derivation process of damping of SRS is given by Dong [18], and need not be repeated here.

2.1. Equivalent moment of inertia

According to the rule of equivalent kinetic energy, the moment of inertia of equivalent crank is given as follows

$$J_c = \sum J_i\left(\frac{\omega_i}{\dot{\theta}}\right)^2 + \sum m_i\left(\frac{v_i}{\dot{\theta}}\right)^2 \tag{3}$$

2.2. Equivalent driving and driven torque

Based on the motor nominal parameter, the driving torque is derived as follows

$$M_d = i_{bb}\eta_{bb}\frac{2\lambda\frac{9550P_e}{n_e}\frac{n_0 - n_e}{n_0}}{[(n_0 - n_e)/n_0]^2} \times \frac{\left(\lambda + \sqrt{\lambda^2 - 1}\right)\omega_0\left(\omega_0 - i_{bb}\dot{\theta}\right)}{\left(\lambda + \sqrt{\lambda^2 - 1}\right)^2\omega_0^2 + \left(\omega_0 - i_{bb}\dot{\theta}\right)^2} \tag{4}$$