Advances in Engineering Software 000 (2015) 1–8

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

The efficiency of dynamic relaxation methods in static analysis of cable structures

M. Hüttner, J. Máca*, P. Fajman

Department of Mechanics, Czech Technical University in Prague – Faculty of Civil Engineering, Czech Republic

ARTICLE INFO

Article history:
Available online xxx

Keywords:
Cable structures
Nonlinear analysis
Dynamic relaxation
Kinetic damping
Viscous damping
Iteration

ABSTRACT

Nine different schemes of dynamic relaxation method are compared in the paper. Schemes with viscous damping and schemes with kinetic damping are used. Kinetic damping with a peak in the middle of the time step and kinetic damping with the parabolic approximation of the peak are considered. They are also used in three different ways of cable approximation. The cable is approximated as a tension bar, a catenary and a parabolic cable element. The efficiency and stability of each method are compared to three selected 3D examples of cable structures and one existing structure. The effect of mass distribution along the structure is also of interest and is studied in the paper.

1. Introduction

The dynamic relaxation method (DRM) is an iterative process that is used to find static equilibrium. DRM is not used for the dynamic analysis of structures; a dynamic solution is used for a fictitious damped structure to achieve a static solution. The method relies on a discretized continuum in which the mass of the structure is assumed to be concentrated at given points (nodes) of the structure. Residual forces (the difference between internal and external forces) are also calculated at these nodes. Nodal displacements are calculated on the basis of Newton's second law of motion, in which residual forces and fictitious variables are used. The diagonal mass matrix and the diagonal damping matrix attenuation are considered and, therefore, the nodal displacement equation may be written for each node separately. And this, in particular, is the main advantage of this method: DRM does not require the assembly and storage of the global stiffness matrix of the structure. DRM is quite suitable for solving large-scale nonlinear problems such as cable structures.

The DRM theory was first described by Day [1]. This theory was further developed by adding a rule for determining the mass to each node [2]. Using kinetic damping is another effective method that was described by Topping [3] and Lewis [4]. Here is also mentioned that the dynamic relaxation method is more stable and more efficient than other stiffness matrix approaches for structures with large degrees of freedom.

The stability of the DRM, the speed of convergence and the CPU time of the solution can be radically affected by the suitable choice of fictitious parameters – i.e. mass and damping. If the masses are too small (in relation to the stiffness of the structure), then the instability of the iteration may occur and the analysis will not converge to the equilibrium state. On the contrary too large fictitious masses lead to the slow and time consuming calculations. The iterative DRM algorithm converges very fast when using the values of damping coefficients close to the critical values. Too large values of damping also lead to the slow and time consuming calculations. For this reason the technique of kinetic damping is employed (with the viscous damping coefficient taken as zero).

The paper compares the effectiveness of different solution strategies for the static analysis of cable structures based on the DRM. The factors having an impact on the stability of the method and the speed of the convergence, such as distribution of the fictitious mass along the structure, using the fictitious damping factor and the choice of the time step are studied in the paper. Three different ways of cable approximation are used – the cable is approximated as a tension bar, a catenary and a parabolic cable element. The efficiency and stability of each solution DRM strategy are compared to three selected 3D examples of cable structures and one existing structure.

This study is a follow-up to study [5]. It has been extended to include the design of the roofing of the Barrandov tram stop in Prague.

2. Approximation of cable

A cable can be approximated as a tension bar, a catenary (several tension bars) and a perfectly flexible element (bending moments along all of its length are equal to zero). Homogeneous material with a constant cross-section throughout its length is assumed in all cases.

* Corresponding author. Tel.: +420 24354500.
E-mail address: maca@fsv.cvut.cz (J. Máca).

http://dx.doi.org/10.1016/j.advengsoft.2015.06.009
0965-9978/

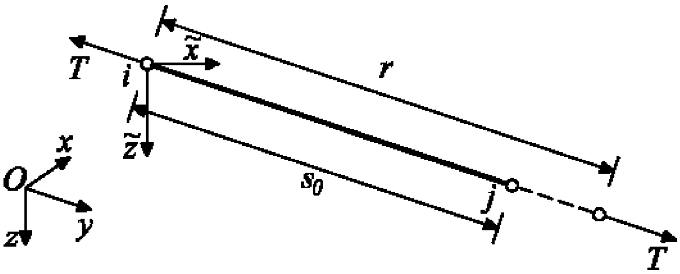

Fig. 1. A bar element.

2.1. Tension bar

The bar connects the endpoints and carries only a positive normal force. The internal force T in one bar element may be calculated using the known Eq. (1) (Fig. 1):

$$T = \frac{EA}{s_0}(r - s_0) \quad (1)$$

where:

- E is Young's modulus of elasticity,
- A is the cross-sectional area,
- r is the distance between two end joints in the chord direction (current length),
- s_0 is the non-elongated length of the element (slack length).

The force T acts as a normal force on the bar. If the force T is negative, then it is set to zero. The deadweight of the strut has been assumed to be concentrated equally at its two end joints.

2.2. Catenary

The basic assumption of this theory is that the behaviour of a cable may be approximated by a few bars. These bars are interconnected by joints and sustain only positive normal forces. The behaviour of individual bars is described in Section 2.1. As found in [6], five bars well enough correctly describe the characteristics of the cable. This approximation enables to describe more precisely the behaviour of individual members (e.g. vertical deflections of the cable).

2.3. Cable element

The basic assumption of the analysis of a flexible elastic cable is that the cable is regarded to be perfectly flexible and is devoid of any flexural rigidity. The load on the cable, which must include at least self-weight, is distributed uniformly along the curve of the cable, which is assumed to be a parabola. The detailed analysis can be found in [7–9].

For the purpose of the study, it is necessary to use an internal force T, which is always positive and whose significance is shown in Fig. 2.

The force T can be calculated iteratively from Eq. (2), as given in [7]:

$$g(T, r, l, c, s_0, Q) = \frac{l^2 T}{2rQ}\left[\ln\left(-\frac{2c}{l} + \frac{rQ}{lT} + \frac{\sqrt{b}}{lT}\right) - \ln\left(-\frac{2c}{l} - \frac{rQ}{lT} + \frac{\sqrt{a}}{lT}\right)\right] + \frac{c}{4rQ}(\sqrt{a} - \sqrt{b}) + \frac{1}{8T}(\sqrt{a} + \sqrt{b}) - s_0 - \frac{T}{EA}\left(\frac{l^2}{r} + \frac{c^2}{r} + \frac{Qr^2}{12T^2}\right) = 0 \quad (2)$$

where:

- l is the horizontal distance between the two end joints,
- c is the vertical separation between the joint j and the joint i (it can be negative),
- r is the distance between two end joints of the cable element,
- s_0 is the slack length of the cable element,
- Q is the resultant of the vertical uniform load q acting vertically along the entire length of a parabolic curved cable, while $Q = qs_0$.

For reasons of clarity, the calculation introduces two more substitutions:

$$a = Q^2r^2 + 4c^2T^2 + 4l^2T^2 + 4crQT;$$
$$b = Q^2r^2 + 4c^2T^2 + 4l^2T^2 - 4crQT.$$

3. Dynamic relaxation

The theory of this method was first described by Day [1]. This theory was further developed and its detailed overview may be found in Barnes [2], Topping [3] or Lewis [4].

3.1. Principle

The basic Eq. (3) of motion for the joint i, the direction j (j corresponds to x, y and z directions) and the time t is:

$$R_{ij}^t = M_{ij} \cdot \dot{v}_{ij}^t + C_{ij} \cdot v_{ij}^t \quad (3)$$

where:

- R_{ij}^t is the residual force at the nodal point i, in the direction j and at the time t,
- M_{ij} is the fictitious mass at the nodal point i and in the direction j,
- C_{ij} is the fictitious damping factor for the nodal point i and in the direction j,
- v_{ij}^t is the velocity at the nodal point i in the direction j and at the time t,
- $\dot{v}_{ij}^t$ is the acceleration at the nodal point i in the direction j and at the time t.

The basic unknowns are nodal velocities, which are calculated from nodal displacements. The discretisation from the timeline with the time step Δt will be performed. During the step Δt, a linear change of velocity is assumed. Acceleration during the step Δt is

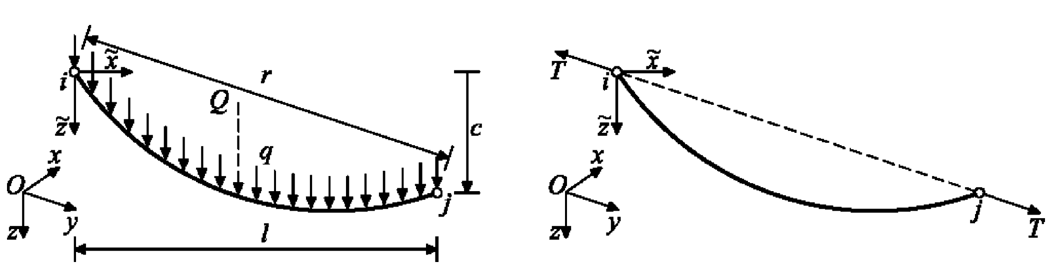

Fig. 2. A cable element. Left – symbols of geometry. Right – tensile force T.

Please cite this article as: M. Hüttner et al., The efficiency of dynamic relaxation methods in static analysis of cable structures, Advances in Engineering Software (2015), http://dx.doi.org/10.1016/j.advengsoft.2015.06.009