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Three-dimensional analysis of a cohesive crack coupled with heat flux through the crack

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ABSTRACT

This paper presents an algorithm for coupling cohesive crack modeling with non-stationary heat flow. Firstly, the nonlinear system of equation, based on global formulations, for such a computational model is derived. The nonlinearity here comes from nonlinear relations in the crack. The relations refer to cohesion forces and to heat flux which both depend of crack opening and additionally are dependent of temperature difference between both sides of the crack. In the paper the discontinuities of displacement field and temperature field are both approximated using XFEM. All the analysis concerning crack surface is performed using local coordinate systems for each integration point. The local coordinate system is two-dimensional for both 2D and 3D analysis. The paper is illustrated with non-stationary thermo-mechanical examples for a domain with propagating crack.

1. Introduction

The aim of the paper is to present the results of three-dimensional coupled thermo-mechanical analysis of heat flow through a solid with a crack. The mechanical model considered in the paper is linear elasticity with a crack inside the body that involves displacement jump between both sides of the crack. The behavior of the crack is described with the help of the cohesive crack model. On the other hand the heat conduction is modeled by the standard non-stationary heat flow. However, the heat flow through the domain is disturbed by the discontinuity of the material.

Computational modeling of the cracking process was usually carried out by the finite element method (FEM) e.g. [1,2]. The element–free Galerkin method (EFGM) was also used in application to crack growth analysis (e.g. [3–6]), or the method based on coupled FEM–EFGM (e.g. [7,8]). Nowadays the methods based on partition of unity method (PUM) are seen as the most accurate technology for crack propagation analysis. In this paper the extended finite element method (XFEM), which is the most widely used in crack analysis problems [9–12]. On the other hand the extended EFGM can also applied for such problems [7,13].

The main aim of the paper is to present coupled analysis of thermoelasticity in the domain with a crack, that is modeled by so called crack surface. A critical review of finite element analysis of solid thermomechanics is performed e.g. in [14]. The thermodynamic foundations of the theory are covered for instance in [15–18]. Recent applications of thermomechanical model coupled with crack or damage can be found for example in [19–23]. In this paper a different approach to the problem is proposed.

The application of the XFEM to the thermo-mechanical crack analysis has been analyzed in recent year in some other papers, e.g. [24–27]. The mentioned papers are limited to the so called adiabatic and isothermal crack models which to not refer to physically motivated problems. The thermo-mechanical crack model presented in this paper is much more general can be used to effectively describe real physical problems. The crack model allow the heat to flow through the crack and the amount of the heat flow depends on both the temperature and displacement jumps. The crack heat model is proposed so that to model the situation when the opened crack is filled with air or other medium.

The physical thermo-mechanical relations between crack faces are described by means of crack local coordinates which are of two-dimensional manner. The coordinates come from the decomposition of crack opening vector into normal and sliding parts. The kind of decomposition was firstly proposed for explicit 3D cohesive crack analysis by Ortiz and Pandolfi in [28]. The idea of decomposing the crack opening vector this way has subsequently been applied successfully in a number of other publications, for example in [29–33]. Te idea of vector decomposition was pursued further to efficiently obtain the tangent stiffness matrix for the Newton–Raphson iterative procedure in implicit analysis in [34].

The paper is based upon Author [35], but the current paper includes the following additional research: (i) accurate mesh updating so that the non-stationary non-linear coupled problem

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with growing crack can effectively be performed, (ii) additional examples are included. In the paper the heat flow through the crack surface is modeled by so-called crack heat flux function. The sort of model for two-dimensional analysis of cracked domain has been applied in [36]. The extension of the approach to three-dimensional case and crack growth analysis is presented in [35]. In this paper the approach is verified and described in tensor notation. The mesh updating procedure connected with the growing crack is added in this paper so that to guarantee the correctness of the final results in the non-stationary problem. Additional examples are added in relation to the presented in [35].

In the following section the mathematical formulation of the coupled thermo-mechanical problem with discontinuities in displacement and temperature fields is presented. Due to fact that the considered problem is nonlinear the linearization procedure is shown in the Section 3. The XFEM approximation including approximation of jumps is presented in Section 4. The crack growth require mesh updating each time when the crack propagates what is shown in Section 5 The algorithm presented in this paper requires integration along the crack surface. For effective calculations the physical relations at crack surface they are done in the crack local coordinates what is described in Section 6. The algorithm is illustrated with examples, Section 7. Finally some conclusions are presented in Section 8.

2. Mathematical model of the problem

The computational model in the paper is derived for fully three-dimensional (3D) case assuming small strains and displacements. The domain under consideration V with outer boundary S is discontinuous at S_d crack surface. The domain, boundary, crack and the local crack two-dimensional local coordinates defined on versors $(\mathbf{n}_d, \mathbf{s}_d)$ are shown in Fig. 1. All the equations connected with the crack are described in the two-dimensional crack local coordinates. The crack normal $\mathbf{n}_d$ comes from geometry of the crack and is known in advance. The second versor $\mathbf{s}_d$ shows the direction of the sliding part of the crack opening vector and have to be evaluated in the calculations. The existence of the crack in the domain, in the coupled thermo-mechanical problem, leads to discontinuities in displacements and in temperature fields.

The analysis of the coupled problem begins with the standard equilibrium equation (momentum balance) that is valid at each point of the considered isotropic solid and for each moment of time:

$$\mathrm{div}\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \qquad \text{in } V \tag{1}$$

where $\mathbf{b}$ is the body force vector. The equation is completed by the following standard boundary conditions:

$$\boldsymbol{\sigma}\mathbf{n} = \mathbf{t} \qquad \text{on } S_\sigma \qquad \mathbf{u} = \hat{\mathbf{u}} \qquad \text{on } S_u \tag{2}$$

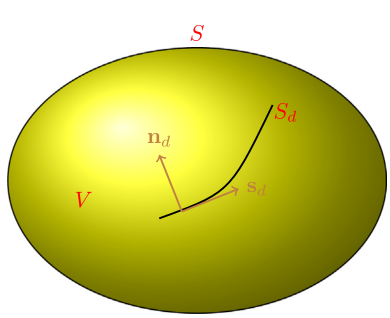

Fig. 1. Domain with inner crack and crack local coordinates.

where $\mathbf{t}$ is the traction forces vector, $\mathbf{u}$ is the displacement vector and $\hat{\mathbf{u}}$ is the prescribed displacement vector.

The additional condition has to be added to the formulation that refers to cohesion tractions along crack surface S_d

$$\boldsymbol{\sigma}\mathbf{n}_d = \mathbf{t}_c \qquad \text{on } S_d \tag{3}$$

where $\mathbf{t}_c$ is the vector of cohesion tractions along the crack surface.

The mechanical model from Eqs. (1)–(3) is reworked into the global weak formulation at time moment $t + \Delta t$ where a test function $\mathbf{v}_\mathrm{u}$ is used

$$\int_V (\nabla_s \mathbf{v}_\mathrm{u}) : \boldsymbol{\sigma}^{t+\Delta t}\,\mathrm{d}V + \int_{S_d} [\![\mathbf{v}_\mathrm{u}]\!] \cdot \mathbf{t}_c^{t+\Delta t}\,\mathrm{d}S - \int_V \mathbf{v}_\mathrm{u} \cdot \mathbf{b}^{t+\Delta t}\,\mathrm{d}V - \int_{S_\sigma} \mathbf{v}_\mathrm{u} \cdot \mathbf{t}^{t+\Delta t}\,\mathrm{d}S = 0 \tag{4}$$

where $[\![\cdot]\!]$ is the discontinuity operator, $\nabla_s \mathbf{v}_\mathrm{u}$ is the symmetric part of $\nabla \mathbf{v}_\mathrm{u}$ tensor. All the details connected with constructing the weak form with the integral along S_d are presented for example in [34]. In the weak equation the vector of cohesion tractions is assumed to be a function of crack opening vector $[\![\mathbf{u}]\!]$.

The discontinuity operator is defined as follow:

$$[\![\mathbf{u}]\!](\mathbf{x}) = \lim_{\lambda \to 0} \mathbf{u}(\mathbf{x} + \lambda \mathbf{n}_d) - \lim_{\lambda \to 0} \mathbf{u}(\mathbf{x} - \lambda \mathbf{n}_d) = \mathbf{u}^+(\mathbf{x}) - \mathbf{u}^-(\mathbf{x}) \tag{5}$$

In this paper the coupled thermo-mechanical model is under consideration. That is why the total strains are be decomposed into elastic $\boldsymbol{\varepsilon}^e$ and thermal $\boldsymbol{\varepsilon}^\Theta$ parts:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^\Theta \quad \text{where} \quad \boldsymbol{\varepsilon}^\Theta = \alpha \Theta \boldsymbol{I} \tag{6}$$

with α the heat expansion parameter, $\Theta = T - T_0$ the relative temperature, i.e. its increase with respect to strain-free (initial, reference) temperature T_0, $\boldsymbol{I}$ the identity matrix.

In the coupled problem analyzed in this paper the non-stationary heat transport problem is considered. The thermal part analysis starts with the energy balance that provides the well–known equation:

$$c\rho\dot{\Theta} + \mathrm{div}\,\mathbf{q} = r, \qquad \text{in } V \tag{7}$$

where ρ is the density, c is the specific heat capacity, $\mathbf{q}$ the heat flux density, r the heat source density. The Eq. (7) has to be completed by standard boundary and initial conditions:

$$\Theta = \hat{\Theta}, \quad \text{on } S_\Theta, \qquad \mathbf{q} \cdot \mathbf{n} = h, \quad \text{on } S_h \tag{8}$$

$$\Theta(t = 0) = 0, \quad \text{in } V \tag{9}$$

The heat flow in the domain is disturbed by material discontinuity along the S_d surface. The heat flow through the crack is modeled by the crack heat flux h_d introduced in [35]. The crack heat flux is related to the heat flux vector by the relation similar to natural boundary condition:

$$\mathbf{q} \cdot \mathbf{n}_d = h_d, \qquad \text{on } S_d \tag{10}$$

The value of the crack heat flux h_d is an additional unknown, besides $\mathbf{t}_c$, that needs to be calculated during the analysis. It is set at the moment that h_d depends on both the crack opening vector $[\![\mathbf{u}]\!]$ and the temperature jump between both sides of the crack $[\![\Theta]\!]$

$$h_d = h_d([\![\mathbf{u}]\!], [\![\Theta]\!]) \tag{11}$$

The weak form of Eq. (7) with the natural boundary condition and condition (10) at time $t + \Delta t$ reads:

$$\int_V v_\Theta c\rho\dot{\Theta}^{t+\Delta t}\,\mathrm{d}\,V - \int_V (\nabla v_\Theta) \cdot \mathbf{q}^{t+\Delta t}\,\mathrm{d}V - \int_{S_d} [\![v_\Theta]\!] h_d^{t+\Delta t}\,\mathrm{d}V = \int_V v_\Theta r^{t+\Delta t}\,\mathrm{d}V - \int_{S_h} v_\Theta h\,\mathrm{d}S \qquad \forall v_\Theta \tag{12}$$

where v_Θ is a test function.

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