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Applicability of the three-parameter Kozeny–Carman generalized equation to the description of viscous fingering in simulations of waterflood in heterogeneous porous media

Nélio Henderson [a,d,*], Juan C. Brêttas [b,d], Wagner F. Sacco [c,d]

[a] Instituto Politécnico, Universidade do Estado do Rio de Janeiro, 28625-570 Nova Friburgo, RJ, Brazil
[b] Escola de Engenharia Industrial Metalúrgica de Volta Redonda, Universidade Federal Fluminense, 27255-125 Volta Redonda, RJ, Brazil
[c] Instituto de Engenharia e Geociências, Universidade Federal do Oeste do Pará, 68135-110 Santarém, PA, Brazil
[d] Thermodynamics and Optimization Group (TOG), Brazil

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ABSTRACT

This article discusses the applicability of the three-parameter Kozeny–Carman generalized equation to trigger immiscible viscous fingers and describe it in fractal heterogeneous porous media, during numerical simulations of waterflood operations in oil reservoirs. For that purpose, for the first time this equation was incorporated into a model that describes immiscible flows of incompressible two-phase fluids in porous media. Results were generated from intensive simulations, and viscous fingers were visualized graphically for three different well patterns, typical of oil fields: Line-Drive, Five-Spot and Inverted Five-Spot. Such results suggest that this generalization of the Kozeny–Carman equation can be used in numerical simulations of oil recovery processes susceptible to hydrodynamic instability phenomena.

1. Introduction

During a secondary petroleum recovery process, conducted under water injection, the formation of immiscible viscous fingering in the oil reservoir is the product of an interfacial instability phenomenon that occurs in porous media, when the oil (the high-viscosity fluid) is displaced by the water (the low-viscosity fluid). In the presence of this instability phenomenon, part of the invading water advances through the porous medium at velocities much higher than those of the average front, bypassing the oil and rapidly approaching the production wells [3]. Thus, the appearance of such immiscible viscous fingers can cause considerable problems for the oil industry, due to poor recovery of hydrocarbons during a waterflood process.

Hence, appropriate phenomenological models, stability analyses of immiscible flows and numerical simulators able to capture the formation of immiscible viscous fingers are very useful tools, not only to predict the viscous fingering but also to assist in the proposal of a possible stabilization process of this phenomenon of hydrodynamic nature, which has (over the past several decades) attracted the attention of many researchers [5,6,8,10,13,17,19,20,22,23].

Studies that discuss the formation of viscous fingers during surfactant injection in heavy oil reservoirs, in the presence and absence of polymers, can be found in the recent works by Yadali Jamaloei et al. [28–32].

In an oil reservoir, when a less viscous fluid displaces a more viscous one, immiscible viscous fingers are triggered primarily by natural geologic heterogeneities, such as the heterogeneities of the real permeability field. In numerical simulation, there are two basic approaches to trigger viscous fingers: (a) small perturbations of the front at time $t = 0$, commonly used in the simulation of flows in homogeneous media, and (b) the effective use of a mathematical model to describe (even in an approximate way) the heterogeneity of the porous material [10]. In the second case, the communities working with reservoir simulations often use *synthetic* permeability fields, generated randomly from a log-normal distribution with a specified variance [12,18].

Recently, Henderson and co-authors proposed the so-called three-parameter Kozeny–Carman generalized (TPKCG) equation [15], which was used in the modeling of heterogeneous permeability fields. More specifically, the TPKCG equation was developed to

* Corresponding author at: Instituto Politécnico, Universidade do Estado do Rio de Janeiro, 28625-570 Nova Friburgo, RJ, Brazil. Tel./fax: +55 (22) 2533 2263.
E-mail address: nelio@iprj.uerj.br (N. Henderson).

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allow the use of a Kozeny–Carman type model to a broad class of porous media with fractal nature, generalizing various models previously reported in the literature, which commonly are employed to describe specific porous media, including oil reservoirs with fractal heterogeneities.

The objective of this work is to verify the ability of the TPKCG equation to trigger immiscible viscous fingers and describe it in a fractal porous medium during numerical simulations of waterflood operations, with adverse mobility ratios. For this, we consider a two-dimensional rectangular porous medium having a random porosity field, with porosity values uniformly distributed in a given interval. Then, the absolute permeability of the porous material is modeled as a function of the porosity, using the TPKCG equation. This approach yields a heterogeneous permeability field, whose fractal nature is described by this mathematical model equipped with three fixed parameters. Assuming this rectangular heterogeneous porous medium as the oil reservoir, in order to observe the effects of well locations on the geometric structure of the viscous fingering, we present results of flows of two immiscible incompressible fluids (water and oil) using three flooding patterns [11], Line-Drive, Five-Spot and Inverted Five-Spot.

Fingering phenomena also can be observed in miscible flows in porous media [9,14,16,25–27]. However, as noted by King et al. [20] the instability of an immiscible flow it is more difficult to model numerically than a miscible flood. In fact, as emphasized by these authors, in an immiscible fingering the so-called Buckley–Leverett mixing zone leads to enhanced stability compared to the corresponding miscible flood. Accordingly, the choice of the present immiscible process incorporates an additional challenge to the use of the TPKCG equation.

The remainder of this paper is organized as follows. In Section 2, we describe the characterization of the porous medium via the TPKCG equation. Section 3 is devoted to the presentation of the mathematical model for immiscible incompressible fluid flows in porous media. In Section 4, we summarize the numerical methods. In Section 5, we report the results. The conclusions are given in Section 6.

2. Design of the porous material

The characterization of porous materials using a Kozeny–Carman type equation necessarily considers the existence of a functional relationship of the form [3]

$$k = f(\phi), \tag{1}$$

where ϕ, the porosity of the material, is the fraction of the bulk volume of the porous medium occupied by voids, and k, the permeability of the material, is the property that determines the ease with which a fluid may be made to flow through the porous medium. In Eq. (1), f summarizes the model used, which generally is a nonlinear function of ϕ, such that $f(\phi) = 0$ if $\phi = 0$.

The TPKCG equation models fractal structures, which are characterized by the existence of fundamental properties between the specific surface (M_b) and the portion of the bulk volume occupied by solid matrix $(1-\phi)$, and between the tortuosity (τ) and the porosity (ϕ), where M_b is defined as the interstitial surface area of the pores per unit of bulk volume of a representative sample of the material, and τ is the ratio of flow-path length to sample-path length. Its formulation assumes that [15]:

(1) The reciprocal of the specific surface admits the fractal scale law

$$\frac{1}{M_b} = C_{1/M_b}(1-\phi)^{-D_{1/M_b}}, \tag{2}$$

where C_{1/M_b} and D_{1/M_b} are, respectively, the fractal coefficient and the fractal exponent of $1/M_b$.

(2) The tortuosity is described by the fractal scale law

$$\tau = C_\tau \phi^{-D_\tau}, \tag{3}$$

where C_τ and D_τ are the fractal coefficient and fractal exponent of τ, respectively.

(3) The porous medium can be modeled as a bundle of n capillary tubes non-necessarily of circular cross-sections, where the flow in this bundle of hydraulic tubes is described by an extension of Hagen–Poiseuille law [3]

$$q = n f_v \frac{R_h^4}{\mu} \frac{\Delta P}{L_h}. \tag{4}$$

In Eq. (4) q is the fluid flow rate in volume per unit time, μ is the viscosity of the fluid, ΔP is the applied pressure difference across the length of the tubes, R_h and L_h denote the hydraulic radius and length of the mean hydraulic tube, and f_v is a shape factor of volume of the tubes.

(4) The hydraulic radius obeys the relation

$$R_h = \frac{\phi}{M_b}. \tag{5}$$

As stated by Henderson et al. [15], such conditions lead to the TPKCG equation, which can be written in the following short form below

$$k = \xi^2 \left[\frac{\phi^{(\zeta+3)}}{(1-\phi)^{2\eta}} \right]. \tag{6}$$

In Eq. (6), the three fractal parameters ξ, ζ and η depend on the fractal coefficients and fractal exponents described in Eqs. (2) and (3), and on the ratio $\sqrt{f_s/f_v}$, where f_s is a shape factor of area of the tubes. The parameters ζ and η are dimensionless quantities, while the parameter ξ has length dimension.

By fitting the model in Eq. (6) to experimental data available in the literature, Henderson et al. [15] were able to prove the potential of the TPKCG equation to describe the permeability–porosity relationships of many natural and industrial materials, including sandstones.

In order to study the formation of viscous fingering, here we consider a two-dimensional heterogeneous porous medium of fractal nature generated from the TPKCG equation, where the three parameters are given by $\xi^2 = 4$ darcy ($4.053\ \mu\text{m}^2$), $\zeta = 1.5$ and $\eta = 0.002$, and the porosity is determined randomly from an uniform distribution on the interval $[0.2, 0.4]$. As shown in Fig. 1, this choice results in a heterogeneous porous medium where the permeability varies roughly between 0.0028 and 0.0648 darcy (0.002837 and 0.065659 μm^2).

3. Two-phase flow equations

In this section, we present the mathematical model used to describe the immiscible flow of an invading fluid (water) plus a resident fluid (oil), which flow together as incompressible two-phase fluids in a porous medium, where the water is considered the wetting phase, while the oil is the nonwetting phase. This flow is described by the mass conservation equations for the wetting phase and for the two-phase fluid, and by Darcy's law for the phase velocities.

In what follows, S denotes the saturation of the wetting phase (the portion of pore volume filled with water), and P is the pressure of the nonwetting phase (oil). The constants μ_w and μ_n are the viscosities of the phases, where "w" and "n" indicate the wetting