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Parallelisation of nonlinear structural analysis using dual partition super elements

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ABSTRACT

This paper presents a new approach of structural domain partitioning for parallel processing based on an interface displacement frame method and using dual partition super-elements. Compatibility and equilibrium between the partitions is ensured at the level of the parent structure, where the dual super-elements handle the two-way communication of interface displacements/forces/stiffness and other analysis-specific entities. The proposed approach can be applied relatively easily to existing monolithic finite element analysis programs, and it offers more freedom to the user with regard to defining partitions, load balancing and isolating parts of the structure of specific interest. Several examples are used in the paper to demonstrate the benefits of the proposed approach.

1. Introduction

The most accurate modelling approach in nonlinear structural analysis is based on the use of 3D finite elements, where the nonlinear material response is represented by tri-axial models, and where complex phenomena, such as bond-slip in reinforced concrete structures, are explicitly captured. This approach, however, is prohibitively expensive and thus typically inapplicable to modelling the overall structural response, where the nonlinear analysis of relatively small structures can require several days of computing time. A possible solution to this problem is to decompose the system into partitions and use parallel computing, which can significantly reduce the wall-clock time required for computations as well as simplify the modelling of large and complex structures in comparison with the monolithic approach [1]. Several approaches have been developed for parallel and partitioned structural analysis including MPI-based parallel finite element approaches [2], finite element tearing and interconnecting (FETI) and other similar methods [3–5], and spectral bisection of meshes [6]. These approaches generally use domain partitioning algorithms such as MeTis [7], JOSTLE [8], and Chaco [9]. The parallel/partitioned approaches are generally limited by the strict conditions implemented at the boundary, prohibiting the use of dimensional coupling or mixed methods such as implicit–explicit integration schemes. Another partitioning approach used is the staggered approach, where different partitions, often representing field-specific models, are computationally treated as isolated entities, and where interactions at the interface are viewed as forcing effects that are communicated between individual components using prediction, substitution and synchronisation techniques [10]. Iterative coupling approaches which enforce equilibrium and compatibility at the interface are generally superior to the standard staggered approach excluding iteration, though this is typically guaranteed only with the availability of the condensed tangent stiffness matrix at the partitioned boundary [11,12].

A new approach of structural domain partitioning for parallel processing is presented in this paper, where a partition child sub-structure in the parent structure/sub-structure is represented by a dual super-element, with one super-element used in the parent process and another used as a wrapper around the partitioned boundary in the child process. Importantly, this approach is hierarchic and can be mapped to hierarchic parallel processing architectures, where any part within a sub-structure can be replaced by a super-element and the replaced part can be modelled separately with all its nodes at the partitioned boundary via the wrapper dual super-element. In addition to the benefits of traditional partitioning approaches, the current approach provides the facility for using mixed methods such as implicit–explicit integration schemes [13,14] as well as dimensional coupling [15,16] between partitions.

A further important benefit of the proposed domain partitioning approach is that it allows the recovery of child partition forces and condensed tangent stiffness matrix at the interface boundary relatively easily via the dual super-element, which can be achieved in a frontal solution method by placing the child super-element at the end of the element ordering list. When all the other elements of the partition are assembled and the associated interior freedoms are eliminated, the remaining equilibrium equations contain the forces and condensed tangent stiffness matrix for the super-element only, which can be communicated to the dual super-element in the parent structure/sub-structure. The parent process treats the partition super-elements similar to other finite elements, providing

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displacements that can be communicated to child processes of the dual super-elements and receiving interface forces and condensed tangent stiffness matrices in return. Thus, this approach is effectively identical in performance to the monolithic approach with high speed-ups due to parallel processing and ease of modelling.

2. Structural decomposition

There are two main types of structural decomposition approaches: one is the domain decomposition approach and the other is displacement frame method [17,18]. The proposed structural decomposition approach is in fact a domain decomposition approach but instead of using linking methods like Lagrange multipliers, it borrows ideas from the displacement frame method.

2.1. Displacement frame method

The proposed domain decomposition approach is based on the interface displacement frame approach, according to which a displacement frame is made to surround the sub-domain. When all internal variables are eliminated [19–25], a stiffness matrix of a new 'element' is obtained that can be used directly with any other element with similar displacement assumptions on the interface. The frame displacements are approximated as:

$$u = Nu^d \tag{1}$$

where u are the displacements on the sub-domain boundary, u^d are the nodal displacements for the interface frame, and N is the shape function of the sub-domain. If Γ_I is the interface boundary, virtual work can be used with discretisation to obtain the nodal forces as:

$$q = \int_{\Gamma_i} N^T t d\Gamma \tag{2}$$

where t are the tractions the sub-domain exerts on the displacement frame. When the tractions are expressed in terms of the frame parameters only, we get:

$$q = Ku^d + f \tag{3}$$

where K is the condensed stiffness matrix of the sub-domain Ω and f are its equivalent boundary forces. Global equilibrium can then be ensured by the condition of zero resultant forces from all the interface frames:

$$\sum_j q^j = 0 \tag{4}$$

With the linearity implied by Eq. (3), this approach is not directly applicable to problems involving geometric and material nonlinearity.

2.2. Proposed approach

Instead of using the discrete frame displacements to approximate the displacements fields on the sub-domain boundary, the proposed domain decomposition approach applies the interface displacement frame to an already discretised sub-domain which caters for nonlinearity as well. Since the sub-domain is already discretised, its boundary nodes are directly assigned to the partition super element.

In order to elaborate the proposed approach further, the terminology of 'parent structure' and 'child partitions' is introduced. Any given structure to be analysed is termed here as the 'parent structure' with some of its parts replaced by 'partition super-elements'. Fig. 1a shows an example of a structure converted into a parent structure, as shown in Fig. 1b, as per this approach. The omitted parts are modelled separately for the purpose of detailed analysis and can use higher order elements or different integration schemes, etc. All the nodes of these partitions that were connected to the rest of the structure are now connected to an element that is the shadow of the super-element used in the parent structure, hence the term 'dual partition super-elements'. It can be seen in Fig. 1c that all the data (e.g. loads, restraints, etc.) related to the part of structure these partitions represent is modelled inside the partitions.

From the perspective of finite element analysis, the behaviour of partitions is represented by the partition super-elements in the parent structure thus completing the structure without any discontinuities, making it possible to apply standard solution procedures. At the partition level, the nodes at the partitioned boundary can be viewed as subject to essential boundary conditions from the parent structure, thus requiring that no further essential boundary conditions are defined at these nodes.

Consider two general sub-domains Ω_1 and Ω_2, subjected to loads P_1 and P_2 respectively with only Ω_2 subjected to some restraints at some of its nodes, as shown in Fig. 2.

Let Ω_1 be designated as the parent structure and Ω_2 as the child partition. If Ω_1 has n degrees of freedom and Ω_2 has m, with h degrees of freedom associated with the dual partition super-elements at the interface boundary, such that $h \leqslant n$ and $h \leqslant m$, the stiffness matrices for the respective sub-domains before condensation are $K_{1(n\times n)}$ and $K_{2(m\times m)}$, respectively.

In nonlinear finite element analysis, K is a tangent stiffness matrix relating the infinitesimal increments of resistance and displacement for one of the discretised sub-domains:

$$\delta R = K\delta d \tag{5}$$

If the sub-domain is subjected to loads P, then the system is in equilibrium if:

$$G = 0 \tag{6}$$

where G is the out-of-balance between load and resistance:

$$G = R - P \tag{7}$$

Lack of equilibrium, hence a non-zero G, is typically remedied via iterative corrections of displacement:

$$\delta d = K^{-1}(-G) \tag{8}$$

with the new displacements obtained from the previous iterative displacements as:

$$d = d_o + \delta d \tag{9}$$

With the new iterative displacements, the resistance is re-evaluated and the iterative process is repeated until Eq. (6) is satisfied.

With the proposed domain decomposition approach, the method remains the same with the difference that h of the m terms of the iterative displacement vector δd_2 of child partition Ω_2 are determined by the parent structure Ω_1 at the partitioned boundary. This is performed at the parent structure level Ω_1 with boundary resistance and condensed tangent stiffness that are obtained at the child partition level Ω_2 from a forward elimination solution stage of Eq. (8). After the h terms of δd_2 are determined at the partitioned boundary, the child process associated with partition Ω_2 continues with the backward substitution solution stage of Eq. (8) so as to obtain the remaining components of δd_2. The child process proceeds to calculate its resistance R_2 and tangent stiffness matrix K_2 for its current displacement d_2, evaluating in the process its out of balance G_2. The forward elimination solution stage of Eq. (8) is then repeated at the child partition level Ω_2 to return the boundary resistance vector of size h and the corresponding $h \times h$ condensed tangent stiffness.

Clearly, the above process systematically decomposes the contribution of the child partition and its interaction with the parent structure. The parent structures Ω_1 utilises the boundary resis-