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Synthesis of modulated–demodulated control systems☆

Ali Bazaei [1], S.O. Reza Moheimani

School of Electrical Engineering and Computer Science, The University of Newcastle Australia, Callaghan, NSW 2308, Australia

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ABSTRACT

We propose systematic design methods for realization of a given LTI compensator with complex poles using a modulated–demodulated control framework. Applicability of the proposed realization methods is established through simulations performed on an undamped resonant plant compensated by a low gain controller to obtain better noise rejection performance. It is demonstrated how a modulated–demodulated structure may reduce the sampling rate in a hybrid control system. In addition, superior robust performance is achieved against variations in baseband parameters using direct reference injection into the modulated–demodulated control systems compared to the indirect injection approach.

1. Introduction

Digital control of high-precision ultrafast mechanisms is a highly challenging task. These systems have extremely fast dynamics, and digital control implementation for them may require specialized hardware that is either inaccessible, or prohibitively expensive. Subsequently, many such systems are operated in an open loop. The emerging high-speed atomic-force microscopes, for example, utilize a nanopositioning stage with extremely fast dynamics. The lateral dynamics of these nanopositioners are often operated in the open loop, due to complications described above (Ando et al., 2003, 2001, 2008; Humphris, Miles, & Hobbs, 2005). The positioning performance of such open-loop systems suffers from parametric uncertainties, unmodeled dynamics, noise, and drift (Bazaei, Yong, & Moheimani, 2012; Daniele, Salapaka, Salapaka, & Dahleh, 1999). There is significant interest in developing new implementation methods for high bandwidth compensators that can control systems with super-fast dynamics.

Modulated–demodulated control system design approach is a method that has important practical applications in controlling fast systems such as radio frequency transmitters (Bode, 1945), MEMS gyroscopes (Chen, M'Closkey, Tran, & Blaes, 2005; Leland, 2003; M'Closkey, Vakakis, & Gutierrez, 2001), rotating gravity gradiometers (Affleck & Jircitano, 1990; Bell, Anderson, & Pratson, 1997; Gerber, 1978), and pulsed jet injection systems (Hendrickson & M'Closkey, 2012). Modulated–demodulated control method was originated from an adaptive control technique that was designed specifically for rejecting sinusoidal disturbances (Bodson, Sacks, & Khosla, 1994; Chen & Paden, 1990). In Chang (1993), the method was used to alleviate the vibrations in lightly damped structures, without using any exogenous modulation signals. In Lau, Quevedo, Vautier, Goodwin, and Moheimani (2007), exogenous modulation signals were incorporated in the control system to further enhance the stability of the closed-loop system. In Lau, Goodwin, and M'Closkey (2004, 2005), performance limitations of a single-channel modulated–demodulated control structure were investigated using an approximate method. In Hendrickson and M'Closkey (2012), a modulated–demodulated structure was introduced for tracking control of sine waves, where the reference parameters are required for *indirect injection* into the structure.

In Bazaei and Moheimani (2013), we presented *direct injection* of the sinusoidal reference into the modulated–demodulated structure for tracking control with no prerequisite on the reference coefficients. In addition, an exact analysis of modulated–demodulated control systems was presented in Bazaei and Moheimani (2013) along with one realization method for second-order LTI compensators with distinct complex poles. However, alternative realization approaches to the second and higher order transfer functions were not discussed in Bazaei and Moheimani (2013) and only a high bandwidth controller was investigated, where effects of measurement noise and digital implementation of

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E-mail addresses: Ali.Bazaei@newcastle.edu.au (A. Bazaei), Reza.Moheimani@newcastle.edu.au (S.O.R. Moheimani).

[1] Tel.: +61 2 4921 5168; fax: +61 2 4921 6993.

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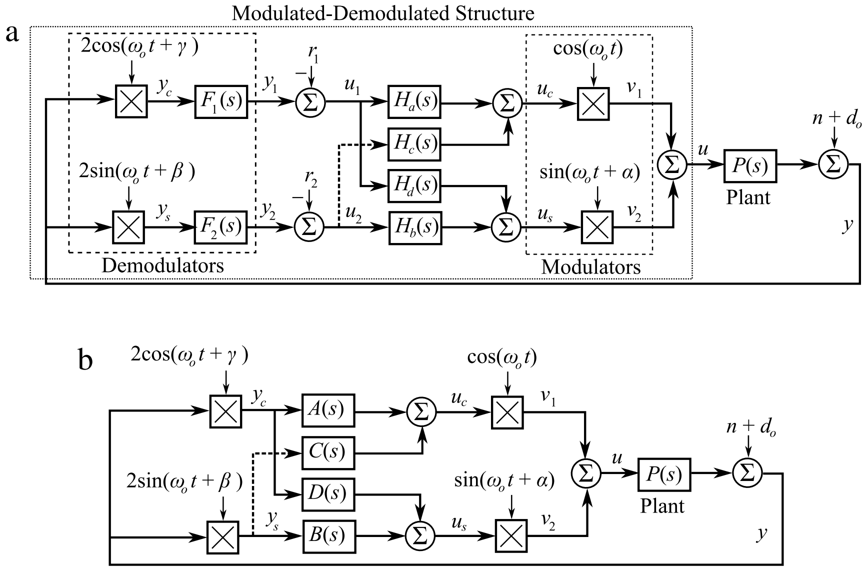

Fig. 1. (a) Schematic diagram of modulated–demodulated control system. (b) Simplified diagram for stability analysis.

controller were ignored. Note that the previously reported modulated–demodulated control systems such as Hendrickson and M'Closkey (2012) and Lau et al. (2007), which were implemented digitally, used sampling rates that were much higher than the plant bandwidth, hindering a valid evaluation of controller discretization effect on the closed-loop performance. Moreover, robustness of modulated–demodulated structures to baseband parameters has not been addressed at moderate and low control bandwidths.

As shown in Fig. 1(a), in a modulated–demodulated control system, controller outputs are modulated by a pair of sine waves before they are applied to the plant. Using low-pass filters F_1 and F_2, the plant output y is then demodulated before being processed by the controller. If the frequency of the sine waves is sufficiently higher than the filter bandwidths, the system will contain two-time-scale signals. Generally, the signals after the low-pass filters and prior to the modulators are slowly varying compared to the plant input and output. The slow time scale in the controller part has the merit of overcoming high frequency design limitations, especially when high bandwidth compensators are required. Such compensators may require extremely high sampling rates for a traditional digital implementation. Using modulated–demodulated strategy, one can use digital methods with limited sampling rates in the baseband to realize high frequency compensators, which may not be implemented directly.

In this paper, we introduce a set of realization methods for implementation of LTI compensators with complex poles, using the modulated–demodulated control system design framework. We start from the results of an exact analysis that decomposes the modulated–demodulated controller into equivalent LTI and LTV (linear-time-varying) components and establishes sufficient conditions under which the modulated–demodulated controller becomes equivalent to an LTI compensator (Bazaei & Moheimani, 2013). Then, a number of realization methods are developed such that any desired transfer function with complex poles can be implemented by modulated–demodulated structures. This allows for a class of LTI controllers, obtained e.g. by traditional design methods, to be implementable using modulated–demodulated structures. A significant advantage of this approach is that the closed-loop stability of the system can be guaranteed a priori.

Compared to previous works on modulated–demodulated control systems, the proposed realization methods establish a systematic approach to determine the requisite low-pass filters, baseband transfer functions, and frequency and phase-angles of the sinusoids used in the modulators and demodulators. Simulation results are presented for simultaneous stabilization of undamped plants and tracking of a sinusoidal reference. The example also demonstrates applicability of the proposed realization methods. In Section 5, we investigate the performance of a recent method reported in Hendrickson and M'Closkey (2012), where two approaches to the injection of sinusoidal references in modulated–demodulated control systems are detailed. Effects of measurement noise and discrete-time implementation of the control system are also considered in the example.

The paper is organized as follows. Section 2 reviews the requisite results on modulated–demodulated control systems, which are used to develop the realization methods. Sections 3 and 4 outline different methods to realize LTI compensators with complex poles using modulated–demodulated structures. In Section 5, applications of the proposed realization methods are demonstrated through a case study. We summarize the results in Section 6.

2. Problem statement

Consider the generalized modulated–demodulated control system depicted in Fig. 1(a). In the absence of the baseband reference signals r_1 and r_2, the filters and the transfer functions in the baseband part can be combined as shown in Fig. 1(b), where:

$$\begin{bmatrix} A(s) & C(s) \\ D(s) & B(s) \end{bmatrix} = \begin{bmatrix} H_a(s) & H_c(s) \\ H_d(s) & H_b(s) \end{bmatrix} \begin{bmatrix} F_1(s) & 0 \\ 0 & F_2(s) \end{bmatrix}. \tag{1}$$

Based on the preliminary analysis reported in Bazaei and Moheimani (2013), we use the following facts about the modulated–demodulated control system.

Theorem 1. *If the baseband reference signals r_1 and r_2 in Fig. 1(a) are zeros, then the modulated–demodulated controller is equivalent to the parallel combination of LTI and LTV systems, as shown in Fig. 2(a), where:*

$$C_m(s) = A_\gamma(s) + B_{\beta-\alpha}(s) + C^*_\beta(s) - D^*_{\gamma-\alpha}(s) \tag{2}$$

and

$$A_\gamma(s) = \frac{e^{j\gamma}A^+ + e^{-j\gamma}A^-}{2}; \qquad A^*_\gamma(s) = \frac{e^{j\gamma}A^+ - e^{-j\gamma}A^-}{2j} \tag{3}$$