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Critical velocity of a uniformly moving load

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ABSTRACT

An analysis of the critical velocity of a load moving uniformly along a beam on a visco-elastic foundation composed of one or two sub-domains is presented. The case study addressed is related to high-speed railway lines. A new formulation of the governing equations in the first order state-space form is proposed for the Timoshenko–Rayleigh beam. Differences in results obtained by Euler–Bernoulli and Timoshenko–Rayleigh beam theories are analysed. It is concluded that, in the case study considered, these differences are negligible. Critical velocities are obtained for load travelling on finite and infinite beams, with and without damping. A new relationship between the viscous damping coefficient and the modal damping ratio is derived and justified. Predictions about critical velocities established in [1] are confirmed numerically for cases not considered in [1], i.e. in cases when the load passes on infinite beams and when damping is considered.

1. Introduction

The response of rails to moving loads is of interest in the area of high-speed transportation. If simple geometries of the track and subsoil are considered, a theoretical concept that is based on the assumption that the track structure acts as a continuously supported beam (the rail) resting on a uniform layer of springs can be introduced. This layer of springs represents the underlying remainder of the track structure, composed of sleepers, ballast, subballast and subgrade. The stiffness of such spring layer along the length of the track is named as the track modulus, also referred to as the modulus of elasticity of the rail support. A single term representing the viscous damping of the foundation is usually added to the governing equations describing transverse vibrations of the rail induced by the moving load.

For such simplified models, analytical or semi-analytical solutions may be derived and thus several concerns related to railway lines can be quickly solved. Other advantages of simplified models are listed as follows: (i) only the main results are available, so they are simple to analyse; (ii) the results usually preserve parameter dependence, allowing for direct sensitivity analysis; (iii) numerical evaluation can be carried out only in places of interest, without following the full time history; (iv) high results precision is ensured within the simplified frame; (v) fast results evaluation is possible. However, due to the simplifying assumptions, it must be stressed that the results obtained reflect only an approximation of the real structural response to the moving load.

Since a considerable amount of studies have been published on this subject, only a few pioneering works are mentioned. Dynamic stresses in the beam structure were first solved by Krylov [2] and later by Timoshenko [3]. Transverse vibrations in a simply supported beam traversed by a constant force moving at a constant velocity were presented by Inglish [4], Lowan [5] and, later on, other solutions have been given by Koloušek [6] and Frýba [7]. In these approaches the deflection field is expressed as an infinite sum of normal modes. Each mode contribution can be obtained by methods of integral transformation, [8].

Solutions for infinite beams were first presented by Timoshenko [9]. The Fourier transform is used for solving the ordinary differential equation. In [10] the effect of the foundation's viscous damping on the response was also discussed. The case of a load variable over time is presented in [11]. The conventional elastic foundation can sustain both compression as well as tension when the beam deforms. The steady state deformation of an infinite beam on a tensionless elastic foundation under a moving load was first studied in [12]. An important comparison between finite and infinite beam characteristics is presented in [13].

When dealing with non-homogeneous supports or foundation stiffness, it is relevant to mention a review by Vesnitskii and Metrikine on transition radiation in mechanics [14]. According to this work, when the load passes at a constant velocity over a discontinuity in the supporting structure, additional vibrations, conventionally referred to as transition radiation, are generated. These vibrations can significantly amplify the beam's deflection field. In [15], transition radiations in elastic systems are analysed. Other analytical studies addressing the foundation stiffness change are presented in [16]. The ones based on the concept of the dynamic stiffness matrix are given in [1,17,18].

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In this paper, the analysis of critical velocity of a load moving uniformly along a beam on visco-elastic foundation is presented. The critical velocity, in the context of this paper, is defined as the load velocity inducing the beam's highest deflections directed downward and/or upward. This velocity is obtained by a parametric analysis during which extreme displacements are determined for each considered velocity. Analyses are carried out on finite as well as infinite beams, on beams for which the foundation is composed of one or two homogeneous sub-domains (see description in Section 2) and with or without damping influence. Free vibrations on finite beams after the load has already left the structure are also analysed. Results related to beams composed of a single sub-domain are obtained by adopting Euler–Bernoulli (E–B) and Timoshenko–Rayleigh (T–R) theories. Due to negligible differences in results, only E–B beams composed of two sub-domains are considered further. Conclusions taken on the beams composed of two sub-domains allow for generalization to several sub-domains.

Several authors presented results obtained on finite simply supported beams [19,20] or clamped beams [21] as the ones that correspond to infinite beams. In these works no care is taken to address the issue of reflected waves. The beam lengths introduced are 30 m, 62.4 m and 32.5 m, respectively, which cannot be considered very long. In this paper it is shown that results obtained on a finite beam on soft elastic foundation cannot be interchanged with results obtained on a corresponding infinite beam. Even a beam longer than specified above (200 m) is considered in this paper. Along with the analysis of critical velocity, the main goal of the study presented in this paper is to identify differences in results of finite and infinite beams in terms of: (i) the maximum displacement gradient with respect to the load velocity; (ii) the effect of transition radiation; (iii) the damping influence.

The consideration of realistic damping behaviour is not a simple task. Total damping should include the material damping and the geometrical (radiation) damping, that is, the geometrical dissipation due to wave propagation into the subsoil. It was proven in [22] that the models represented by a low number of parameters like the one used in this paper cannot correctly represent the geometrical dissipation. Material damping should encompass both internal friction in the beam as well as damping of the geomaterial representing the foundation. According to experiments, material damping of geomaterials is frequency independent [23], thus it should be modelled as hysteretic damping. Often viscous damping is considered instead, because it leads to a convenient form of the equation of motion [23], but then the energy loss per cycle is dependent upon the excitation (or response) frequency. Due to several simplifying assumptions already adopted, a more realistic damping model would not improve accuracy significantly, therefore only viscous damping is considered. The value of the critical viscous damping coefficient related to infinite beams is very often introduced in an approximate way [24]. The correct formulation for infinite Euler–Bernoulli (E–B) beams is given in [7], and a new (not yet published) formula is derived in this paper for the critical damping of infinite Timoshenko–Rayleigh (T–B) beam.

Along the developments presented in this paper, new formulations are given for: (i) the first order state-space form of T–R beam; and (ii) for the relation between the viscous damping constant and the modal damping ratio assuring the same level of damping in lightly damped finite beam structures.

The critical velocities determined for the case study considered are still unattainable by nowadays trains. Nevertheless, results presented in this paper have practical importance because they show values of extreme displacements as a function of velocity. Especially augmented displacements directed upward, that aggravate track deterioration, should be avoided in railway applications.

The paper is organized in the following way: in Section 2, a general description of the problem and the simplifying assumptions are stated. In Section 3, the case study is defined. In Section 4, finite and infinite beams composed of one sub-domain are analysed under the assumption of E–B and T–R beam theory. In Section 5, finite and infinite beams composed of two sub-domain are analysed under the assumption of E–B beam theory. In Section 6, numerical results are presented. Conclusions are drawn in Section 7.

2. General description of the problem

A uniform motion of a constant single vertical force along a horizontal beam on a linear visco-elastic foundation is assumed. The foundation is modelled as distributed springs and dashpots. The beam is homogeneous with a uniform cross section made of a linear elastic material and its damping is proportional to the velocity of vibration. The load's inertia is neglected.

Finite simply supported and infinite beams will be addressed. The foundation will be composed of one or two sub-domains with uniform properties. In Fig. 1, a finite simply supported beam on a foundation composed of two sub-domains is shown. P stands for the moving force, v is its constant velocity, x and w are spatial coordinate and vertical deflection. The deflection is assumed positive when oriented downward and is measured from the equilibrium position, when the beam is only loaded with its own weight. At zero time ($t = 0$) the load is located at the origin of the spatial coordinate x.

The critical velocity of the single load will be addressed, examined and compared for finite and infinite beams on a homogeneous foundation (foundation composed of a single sub-domain) and on a foundation composed of two sub-domains. Conclusions drawn are possible to extend to situations with more foundation sub-domains. For the sake of simplicity, the term "sub-domain" will be used not only for the foundation, but also for the corresponding beam structure. Transverse vibrations induced by the load are solved by the normal-mode analysis. The natural frequencies are obtained numerically exploiting the concept of the global dynamic stiffness matrix. This ensures that the frequencies obtained are accurate. For infinite beams composed of two sub-domains the method described in [17] is adopted. Results on homogeneous infinite beams are obtained according to [7,24].

In this context, it is necessary to review previous works. In [1], the load critical velocity on undamped finite beams composed of

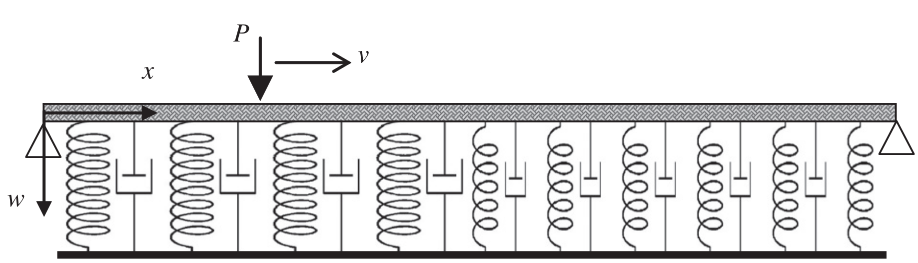

Fig. 1. Simply supported beam on a foundation composed of two sub-domains.