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Brief paper

Bounded-error identification for closed-loop systems[☆]

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ABSTRACT

This paper presents a scheme for the identification of a system which operates in closed-loop and in the presence of bounded output disturbances. Two algorithms are proposed to solve this identification problem. The first algorithm is an Optimal Bounding Ellipsoid (OBE) type algorithm. This first algorithm is analyzed and sufficient conditions for stability and convergence are established. Relaxation of these conditions leads to a second identification algorithm. The implementation of that second algorithm is realized in an iterative scheme. A numerical example is provided to show the efficiency of the scheme.

1. Introduction

1.1. The considered identification problem

This paper is devoted to the study of a Set Membership Identification (SMI) algorithm for a dynamic SISO system operating in the presence of feedback. Here the system is assumed to be parameterized by a discrete-time transfer function $G^*(q)$ such that the closed-loop behavior of the system satisfies $\begin{cases} y_t = G^*(q)u_t + w_t \\ u_t = r_t - C(q)y_t \end{cases}$, it follows

$$y_t = \frac{G^*(q)}{1 + G^*(q)C(q)} r_t + v_t \tag{1}$$

with $v_t = \frac{1}{1+G^*(q)C(q)} w_t$. $C(q)$ is the linear controller (supposed to be known) and r_t an exogenous input signal. The sequence w_t is not observable but is known to be bounded in the ℓ_1 norm: $|w_t| \leq \delta_w$. Through the closed-loop w_t produces the bounded sequence v_t such that

$$|v_t| \leq \delta_v. \tag{2}$$

It represents noise measurements, state disturbances or modeling inaccuracies brought back on the output of the closed loop.

This closed-loop SMI problem occurs when open-loop experiment is prohibited or has no meaning (safety, stability, economical reasons, efficiency of operation, etc.) and when the diversity of the components on w_t is such that its probability density function is unknown.

1.2. Prior work

The identification of closed-loop systems has received much interest for the last decades (see e.g. Agüeroa, Goodwin, & Van den Hof, 2011 and Forssell & Ljung, 1999) and three specific groups of methods can be distinguished: (1) the direct approaches in which the identification is performed as in an usual open-loop context (Chiuso, 2006, Chiuso & Picci, 2005 and references therein), (2) the indirect approaches which are mainly based on an analysis of the control system sensitivity function using the system output and an external excitation input (see Forssell & Ljung, 1999, Gilson & Van Den Hof, 2005, Van Den Hof & Schrama, 1993, Van Overschee & De Moor, 1997) and (3) the joint input–output approaches which use the system input–output behavior together with an external excitation input (see Katayama & Tanaka, 2007, Verhaegen, 1993). These methods aim at providing an unbiased model of the plant in the stochastic noise assumption. If the only information about the noise is its instantaneous bound, these methods are not able to efficiently identify the system.

SMI methods are the identification methods introduced to deal with system identification when the noise is assumed to be unknown but bounded. Here we consider noise bounded in the ℓ_1 norm. Unlike the other identification approaches, which provide an estimate, SMI methods propose the estimation of a feasible parameter set i.e. a model set compatible with all the available information. There are two main possible structures for the design of

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this feasible parameter set: a polytope or an ellipsoid. In this paper we shall investigate a particular type of ellipsoidal algorithm: the Optimal Bounding Ellipsoid (OBE) type algorithms. The reason is that their computational complexity is low and they are appropriated to handle the identification problem in presence of bounded disturbances. Some contributions have been presented in Canudas-De-Wit and Carrillo (1990), Dasgupta and Huang (1987), Fogel and Huang (1982), Pouliquen, Pigeon, and Gehan (2011) and Tan, Wen, and Soh (1997).

In the above methods very few of them are devoted to the direct identification problem expressed as $|y_t - \widehat{G}(q)u_t| \le \delta_w$ with $\widehat{G}(q)$ an IIR filter and δ_w fixed in advance. Among them, some are only suitable for the identification of stable systems (Cerone, 1993a,b; Clement & Gentil, 1990; Ferreres & M'Saad, 1997; Pouliquen et al., 2011) and others have a high computational complexity (Cerone, 1993b; Cerone, Piga, & Regruto, 2012). Above all, none of them ensures the estimation of a model which stabilizes the closed-loop, this is however an essential elementary property.

In this paper, to get around these difficulties, we consider the indirect identification problem expressed as $\left|y_t - \frac{\widehat{G}(q)}{1+\widehat{G}(q)C(q)}r_t\right| \le \delta_v$ with δ_v fixed in advance. In the above challenging problem, the number of alternatives is very limited. One alternative is to use an SMI algorithm in an indirect two steps approach: 1—the transfer function $\frac{G^*(q)}{1+G^*(q)C(q)}$ between r_t and y_t is identified, 2—$G^*(q)$ is retrieved from the identified transfer function under the condition that the controller is linear and known. This approach leads however to a high order model and the use of a model reduction step would probably not maintain the property $\left|y_t - \frac{\widehat{G}(q)}{1+\widehat{G}(q)C(q)}r_t\right| \le \delta_v$. This paper consists in the development of a new alternative which alleviates some of the issues of the previous methods.

1.3. Contributions of this paper

The first key idea in our development is the proposition of a first algorithm using an OBE type algorithm together with the closed-loop Output Error (CLOE) parametrization introduced in Landau and Karimi (1997). Such a parametrization is not linear in the parameter vector. This non-linear effect impacts the stability analysis and a main contribution is the establishment of stability and convergence conditions of the algorithm. The second key idea in our development is the relaxation of the previous stability conditions via a second identification algorithm. This leads to the estimation of a model such that $\left|y_t - \frac{G(q)}{1+G(q)C(q)}r_t\right| \le \delta_v$ without over-parametrization. The current paper completes the work presented in Pouliquen, Gehan, Pigeon, and Frikel (2012).

The paper is organized as follows: the identification problem is formulated in Section 2. In Section 3, two identification algorithms are presented. The first one is described and analyzed in detail in Sections 3.1 and 3.2, the second one is introduced in Section 3.3. The proposed algorithms have been tested on a numerical application, results are given in Section 4. Section 5 concludes the paper. Appendices contain most of the proofs.

2. Problem formulation

Consider the transfer function $G^*(q)$ parameterized as

$$G^*(q) = q^{-d}\frac{B^*(q)}{A^*(q)} \tag{3}$$

with $\begin{cases} B^*(q) = b_0^* + b_1^* q^{-1} + \cdots + b_{n_b}^* q^{-n_b} \\ A^*(q) = 1 + a_1^* q^{-1} + \cdots + a_{na}^* q^{-na} \end{cases}$. q^{-1} is the delay operator, d is the delay, n_a and n_b the degrees of respectively $A^*(q)$ and $B^*(q)$. Let us denote $\theta^* \in \mathbb{R}^n$ the parameter vector with $n = n_a + n_b + 1$ the number of parameters: $\theta^{*\,T} = \begin{pmatrix} \cdots & a_i^* & \cdots & b_i^* & \cdots \end{pmatrix}$. Making use of the CLOE parametrization, $y_t = \frac{G^*(q)}{1+G^*(q)C(q)}r_t + v_t$ can be re-expressed as $y_t = \hat{y}_t + v_t$ where $\hat{y}_t$ is determined by $\hat{y}_t = \phi_t^T\theta^*$ with $\phi_t = \begin{pmatrix} \cdots & -\hat{y}_{t-i} & \cdots & \hat{u}_{t-d-i} & \cdots \end{pmatrix}$and $\hat{u}_{t-d-i} = r_{t-d-i} - C(q)\hat{y}_{t-d-i}$.

Objective: Given the degrees n_a and n_b, the aim of this paper is to present an identification scheme in order to find an estimate $\widehat{\theta}$ for θ^*. The transfer function $\widehat{G}(q)$ parameterized by $\widehat{\theta}$ must satisfy

$$\left|y_t - \frac{\widehat{G}(q)}{1+\widehat{G}(q)C(q)}r_t\right| \le \delta_v. \tag{4}$$

This must be done by using the available data $\{r_t, y_t\}$, the knowledge of the controller $C(q) = \frac{R(q)}{S(q)}$ and the upper bound δ_v.

The estimate for θ^* at the instant t is denoted $\hat{\theta}_t$. For this current time t, $\hat{y}_t$ is replaced by its a priori and a posteriori estimates $\begin{cases} \hat{y}_{t/t-1} = \hat{\phi}_t^T\hat{\theta}_{t-1} \\ \hat{y}_{t/t} = \hat{\phi}_t^T\hat{\theta}_t \end{cases}$. The pseudo linear regression vector ϕ_t is substituted by $\hat{\phi}_t$ which is simply obtained by replacing the unknown component $\hat{y}_{t-i}$ by its a posteriori estimate $\hat{y}_{t-i/t-i}$ and $\hat{u}_{t-d-i}$ by its a posteriori estimate $\hat{u}_{t-d-i/t-d-i}$:

$\hat{\phi}_t^T = \begin{pmatrix} \cdots & -\hat{y}_{t-i/t-i} & \cdots & \hat{u}_{t-d-i/t-d-i} & \cdots \end{pmatrix}$

with $\hat{u}_{t-d-i/t-d-i} = r_{t-d-i} - C(q)\hat{y}_{t-d-i/t-d-i}$.

The a priori and a posteriori prediction errors are derived from the previous definitions in the following form: $\begin{cases} \epsilon_{t/t-1} = y_t - \hat{y}_{t/t-1} \\ \epsilon_{t/t} = y_t - \hat{y}_{t/t} \end{cases}$. Let us notice that the a posteriori prediction error $\epsilon_{t/t}$ can be easily expressed as:

$$\epsilon_{t/t} = \frac{S(q)}{A^*(q)S(q) + q^{-d}B^*(q)R(q)}\hat{\phi}_t^T\tilde{\theta}_t + v_t \tag{5}$$

where $\tilde{\theta}_t = \theta^* - \hat{\theta}_t$ denotes the parameter error vector.

3. Identification algorithms and analysis

3.1. The CLOE-OBE (closed-loop Output Error-OBE) algorithm

From (1) and (2) the parameter vector θ^* belongs to the set defined by $\bigcap_i^t \mathcal{S}_i$ with $\mathcal{S}_t = \left\{\theta \in \mathbb{R}^n, |y_t - \phi_t^T\theta| \le \delta_v\right\}$. The first OBE algorithm to be presented builds on that property in the sense that its aim is to find a parameter vector $\hat{\theta}_t$ center of an ellipsoid $\mathcal{E}_t$ such that $\mathcal{E}_t \supset \bigcap_i^t \widehat{\mathcal{S}}_i$ where $\widehat{\mathcal{S}}_t$ is the observation set defined by $\widehat{\mathcal{S}}_t = \left\{\theta \in \mathbb{R}^n, |y_t - \hat{\phi}_t^T\theta| \le \delta\right\}$. δ is a user defined bound which has to be specified taking into account the bound δ_v. Given $(y_t, \hat{\phi}_t)$, $\widehat{\mathcal{S}}_t$ is the set of all possible θ which are consistent with the chosen bound δ. An important property of this observation set is given in the following theorem. In this theorem $\|.\|_1$ is the l_1 induced norm.

Theorem 1. *Consider a parameter vector $\hat{\theta}_t$ such that $\hat{\theta}_t \in \widehat{\mathcal{S}}_t$. Assume that $G^*(q)$ and δ are such that:*

$$\left\|1 - \frac{A^*(q)S(q) + q^{-d}B^*(q)R(q)}{S(q)}\right\|_1 < 1 \tag{6}$$

$$\delta \ge \frac{\left\|\frac{A^*(q)S(q)+q^{-d}B^*(q)R(q)}{S(q)}\right\|_1}{1 - \left\|1 - \frac{A^*(q)S(q)+q^{-d}B^*(q)R(q)}{S(q)}\right\|_1}\delta_v. \tag{7}$$

Then

$$\theta^* \in \widehat{\mathcal{S}}_t. \quad \blacksquare \tag{8}$$

This theorem states that the ability to find the true parameter vector inside $\widehat{\mathcal{S}}_t$ depends on one condition on $G^*(q)$ and one condition on δ. From (7) the choice on δ depends not only on the known controller $C(q)$, and on the known bound δ_v but also on the unknown polynomials $A^*(q)$ and $B^*(q)$. In Section 3.3 a filter will be introduced so as to relax these hard conditions.