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Brief paper

Absolute stability of the Kirchhoff string with sector boundary control☆

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ABSTRACT

This paper addresses the stability problem of the nonlinear Kirchhoff string with nonlinear boundary control. The nonlinear boundary control is the negative feedback of the transverse velocity of the string at one end, which satisfies a sector constraint. Employing the integral type multiplier method, we establish explicit absolute exponential stability of the Kirchhoff string system.

1. Introduction

The stabilization problem of distributed parameter mechanical systems in engineering has attracted much attention, see e.g. Liu and Krstic (2001), Morgül (1990). Especially, stabilization of string or beam systems has been extensively studied by several authors in the last three decades, such as Fung, Wu, and Wu (1999), Li, Aron, and Rahn (2002), and Wang and Li (2004). In particular, boundary feedback stabilization of string and beam systems has become an important research area (He, Ge, Howa, Choo, & Hong, 2011; Smyshlyaev, Guo, & Krstic, 2009).

As a typical distributed parameter infinite-dimensional system, the Kirchhoff string has been spotlighted for a half century from the view of both mathematics and engineering (see Arosio and Garavaldi (1991), Miranda and Jutuca (1999) and references therein). Mathematically, challenging issues are the well-posedness of solutions in the global or local sense and the dynamical behaviors of solutions under the various data conditions. For example, Ono (1997) proved global existence, decay estimates and blow up results for a degenerate non-linear wave equation of the Kirchhoff type with strong dissipation, and Taniguchi (2010) obtained the existence of a local solution to a weakly damped Kirchhoff equation with the damping term and the source term. On the other hand, the stabilization and convergence of Kirchhoff strings under external control is also an important subject for the control practice of mechanical systems. The stabilization problem with boundary velocity feedback was studied by Shahruz and Krishna (1996). In the literature, a linear boundary velocity feedback control law was proposed, and it was shown that the system under the proposed control law is exponentially stable by constructing an energy-like function, which is motivated by the spirit of the Lyapunov direct method. Furthermore, a similar approach was used in Shahruz (1997) and Li, Hou, and Li (2008) to prove the stability of various nonlinear axially moving strings controlled by the linear boundary feedback.

On the other hand, the absolute stability and the input-to-state stability theory for the infinite-dimensional nonlinear feedback system, which is a feedback interconnection of an infinite-dimensional linear system and a sector-bounded nonlinearity, has been well studied in recent years (Curtain, Logemann, & Staffans, 2004; Jayawardhana, Logemann, & Ryan, 2008; Logemann & Ryan, 2000). This paper investigates the absolute stability problem for nonlinear Kirchhoff strings with a boundary feedback control law, which satisfies the sector condition. The feedback control law provides mechanical power dissipation with an external force.

In fact, proving the stability by showing the decreasing property of energy function is a well-known approach in lumped

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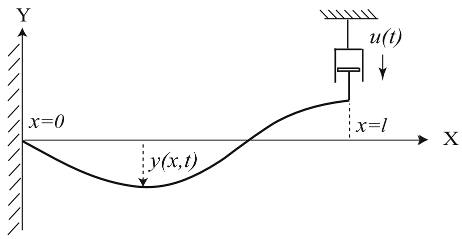

Fig. 1. Schematic of the nonlinear Kirchhoff string with boundary control.

parameter finite dimensional systems. This approach, so-called passivity-based control (Khalil, 2001), claims that a dynamical system might be stabilized by output feedback control, usually velocity feedback in rigid mechanical systems, if the system is passive with respect to the external energy supply rate. In this case, the stability is guaranteed by the Lyapunov direct method. It is not difficult to show the passivity for the nonlinear Kirchhoff string forced at the boundary. Therefore, if the external force acts as negative velocity feedback physically, which means that negative mechanical power is supplied to the mechanical system, stability can be expected based on the passivity. However, the bottleneck in reaching the asymptotic stability from the passivity is the fact that the convergence of the infinite-dimensional state cannot be easily shown from the passivity inequality.

The main contribution of this paper is to show that the system is absolutely stable if the feedback control law is continuous and is not limited to smoothness, and the static gain function satisfies a sector condition. By using the multiplier method based on an integral inequality, we prove the exponential stability result for this system. To validate the control approach, a numerical example will be demonstrated where the nonlinear distributed parameter infinite-dimensional equation is solved by applying the finite element method.

2. Preliminaries

Fig. 1 shows an elastic string with a controller at the right boundary, where $y(x,t)$ represents the transversal displacement of the Kirchhoff string at spatial coordinate x, and time t with the mass density ρ, the Young modulus ε, the cross-section area h, and the initial axial tension τ_0. The left boundary $x=0$ is fixed, and the right boundary $x=l$ permits a transversal movement of the string under a control force $u(t)$.

It is well known that the model of the nonlinear Kirchhoff string in Fig. 1 is represented by the following equations (Kirchhoff, 1883):

$$y_{tt}(x,t) = \left[a + b\int_0^l y_x^2(x,t)dx\right] y_{xx}(x,t) \tag{1}$$

for all $x \in (0,l)$ and $t \geq 0$, with the boundary conditions

$$y(0,t) = 0, \tag{2}$$

$$\left[a + b\int_0^l y_x^2(x,t)dx\right] y_x(l,t) = u(t), \tag{3}$$

for all $t \geq 0$, where $a = \frac{\tau_0}{\rho h} > 0$, $b = \frac{\varepsilon}{2\rho l} \geq 0$. Moreover, the tension in the string represented by Eq. (1) is given by

$$\mathcal{T}(t) = a + b\int_0^l y_x^2(x,t)dx. \tag{4}$$

The boundary condition (3) represents the balance of the transversal component of the tension in the string and the control input u, which is applied transversally at the right boundary $x=l$.

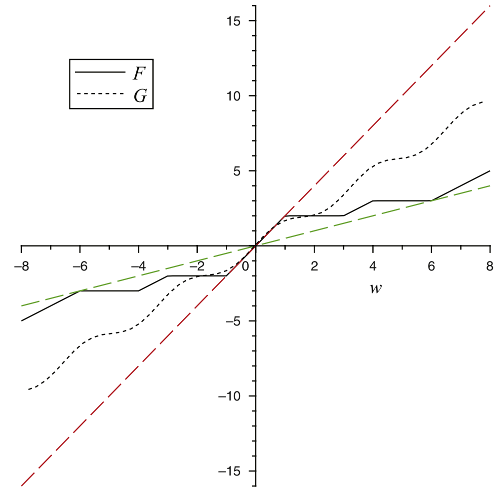

Fig. 2. A linear sector with $L_1 = 1/2, L_2 = 2$.

The control input u in Eq. (3) is commonly known as the boundary control (Shahruz & Krishna, 1996). The measurement output of the system is $y_t(l,t)$. For convenience of reference, the governing equation, the initial state, and the boundary conditions are put together as follows: for all $x \in (0,l)$ and $t \geq 0$,

$$\begin{cases} y_{tt}(x,t) = \mathcal{T}(t)y_{xx}(x,t), & \text{(a)} \\ y(0,t) = 0, & \text{(b)} \\ \mathcal{T}(t)y_x(l,t) = u(t), & \text{(c)} \\ y(x,0) = f(x), & \text{(d)} \\ y_t(x,0) = g(x), & \text{(e)} \end{cases} \tag{5}$$

where $f(x)$ and $g(x)$ are the initial displacement and velocity of the string, respectively.

In this paper, we consider a boundary velocity feedback control given by

$$u(t) = -F(y_t(l,t)), \tag{6}$$

where the static nonlinearity F is non-decreasing and Lipschitz continuous function satisfying the following sector constraint condition

$$F(0) = 0 \quad \text{and} \quad L_1 \leq \frac{F(w)}{w} \leq L_2, \quad \forall w \in R \setminus \{0\}, \tag{7}$$

for given positive constants L_1 and L_2. Fig. 2 shows two examples of such a function that satisfies the constraint condition with $L_1 = 1/2, L_2 = 2$. The examples show that the feedback gain can be chosen to be non-linear, with the slope depending on the velocity. The function F is a typical quantized feedback control law in Fig. 2. The Kirchhoff string system (5) combined with the feedback law (6) becomes a closed-loop system. The main problem addressed in this paper is to provide a stability analysis result for the closed-loop feedback control system (5)–(6). The absolute stability of the system will be proven by constructing an energy function that decays exponentially along any solution of the partial differential functions.

For notational convenience, instead of $y_x(x,t)$ and $y_t(x,t)$, y_x and y_t will be used, with similar abbreviations employed subsequently. We first state the global existence results for the closed-loop system (5)–(6). Let $V = \{y \in H^1(0,l) : y(0) = 0\}$ and $W = \{y \in H^2(0,l) : y(0) = 0\}$. Let $\|\cdot\|$ denote the $L^2(0,l)$ norm.

Proposition 2.1. *Assume that the static feedback nonlinearity F satisfies sector constraint condition (7). If the initial date satisfies $(f,g) \in W \times V$ and the compatibility condition $M(\|f_x\|^2)f_x(l) = -F(g(l))$ with $M(s) := a + bs$, then the closed-loop system (5)–(6) admits a global (weak) solution, such that, for all $T > 0$, $y \in L^\infty([0,T];V)$; $y_t \in L^\infty([0,T];V)$; $y_{tt} \in L^\infty([0,T];L^2(0,l))$.*