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Brief paper

Containment control of multi-agent systems in a noisy communication environment☆

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ABSTRACT

The containment problem of first-order and second-order integral multi-agent systems with communication noises is investigated in this paper. Motivated by the previous work, a time-varying gain $a(t)$ is introduced to attenuate the noise effect. It is proved that the proposed protocol can solve the mean square containment problem if the following conditions hold: (1) the communication topology graph has a spanning forest whose roots are leaders of the multi-agent system; (2) $\int_0^\infty a(t)dt = \infty$; (3) $\lim_{t\to\infty} a(t) = 0$. Moreover, if $a(t)$ is uniformly continuous, these conditions are necessary for ensuring the mean square containment. In addition, the proposed protocol is also extended to allow the case where each agent has its own gain $a_i(t)$. It is proved that the mean square containment problem can still be solved if all agent-dependent gains are infinitesimals of the same order as time goes to infinity. Finally, two simulation examples are provided to demonstrate the effectiveness of the proposed protocols.

1. Introduction

In the past decade, the containment control of multi-agent systems (MASs) becomes one focal topic due to its great potential in scientific and engineering fields. Fruitful results have been obtained in the field such as Cao, Ren, and Egerstedt (2012), Liu, Xie, and Wang (2012) and Notarstefano, Egerstedt, and Haque (2011), to name a few. In Notarstefano et al. (2011), it was proved that the followers could be convergent to some points in the convex hull spanned by the leaders if the undirected topology graphs are jointly connected. In Cao et al. (2012), necessary and sufficient conditions were presented for ensuring the containment, that is: for each follower there exists at least one leader which has a directed path to the follower. Liu et al. (2012) investigated the containment problem of MASs with both stationary and dynamic leaders and employed the concept of spanning forest to describe the communication topology graph. It is noted that the MASs are assumed to work in an ideal communication environment in most existing papers. However, in practice, some communication constraints (e.g. communication noises (Cheng, Wang, Hou, Tan, & Cao, 2013; Huang & Manton, 2009; Wang, Cheng, Hou, Tan, Zhou, et al., 2013) and delays (Hu, Chen, & Li, 2011; Hu & Hong, 2007; Lin & Ren, 2014)) are unavoidable. This paper is devoted to studying the containment problem with communication noises.

According to the studies on the consensus problem, it is found that the traditional algorithms designed for the noise-free communication cannot solve the consensus problem of MASs with communication noises. It is well known that the containment problem of MASs is an extension to the leader-following consensus problem of MASs. It is therefore reasonable to believe that existing containment protocols cannot be directly applied to the noisy environment either. Some noise attenuation techniques should be employed to deal with the noise's effect, which are briefly reviewed in the following part. In Huang and Manton (2009), a stochastic approximation type gain $a(t)$ ($\int_0^\infty a(t)dt = \infty$ and $\int_0^\infty a^2(t)dt < \infty$ in the continuous time domain and $\sum_{t=0}^\infty a(t) = \infty$ and $\sum_{t=0}^\infty a^2(t) < \infty$ in the discrete time domain) was first introduced to the consensus protocol. The basic idea is to reduce the effect of communication noises by multiplying it with a time decreasing gain. In Li and Zhang (2009), necessary and sufficient conditions on the mean square consensus were given for continuous-time first-order integral MASs with communication noises. Wang and Zhang

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(2009) proved that the almost sure consensus problem can also be solved by the same protocol proposed in Li and Zhang (2009). In Cheng, Hou, Tan, and Wang (2011), a consensus protocol was proposed for continuous-time second-order integral multi-agent systems with communication noises. Necessary and sufficient conditions for mean square average consensus were obtained as well. Extensions to the sampled-data based control and to the general linear MASs were made in Cheng, Hou, and Tan (2014), Cheng et al. (2013), respectively. Besides the above leaderless consensus results, the leader-following consensus problem of MASs with communication noises was also studied in Hu and Feng (2010), Ma, Li, and Zhang (2010), Wang, Cheng, Hou, Tan, Liu, et al. (2013) and Wang and Zhang (2009), where the time-varying gain played an important role in reducing the noise's effect. Since the leader-following consensus problem is a bridge connecting the leaderless consensus problem and the containment problem, the containment problem in the noisy communication environment is expected to be solved by the technique of time-varying gain too.

An early attempt on this topic was made in Tang, Huang, and Shao (2012), where the stochastic containment problem of discrete-time first-order integral MASs was solved by using the stochastic approximation type gain. In this paper, two containment protocols are proposed for continuous-time first-order and second-order integral MASs with communication noises, respectively. A time-varying gain $a(t)$ is employed to attenuate the noise's effect. It is proved that under the proposed protocols, all followers' positions are convergent in mean square to certain points in the convex hull spanned by the leaders' positions if the following three conditions hold: (1) the communication topology graph has a spanning forest whose roots are exactly the leaders of MASs; (2) $\int_0^{\infty} a(t)dt = \infty$; (3) $\lim_{t\to\infty} a(t) = 0$. Moreover, if $a(t)$ is uniformly continuous, these three conditions are also necessary. Since $\int_0^{\infty} a^2(t)dt < \infty$ implies $\lim_{t\to\infty} a(t) = 0$ when $a(t)$ is uniformly continuous, the conditions on $a(t)$ in this paper are relatively weaker than the stochastic approximation type gain employed in Cheng et al. (2011), Hu and Feng (2010), Ma et al. (2010) and Tang et al. (2012). Furthermore, this paper also proposes a modified containment protocol where each agent has its own gain $a_i(t)$. This protocol setting overcomes the limitation in Cheng et al. (2014, 2011), Hu and Feng (2010), Ma et al. (2010), Tang et al. (2012), Wang, Cheng, Hou, Tan, Liu, et al. (2013) and Wang and Zhang (2009), where all agents need to share a common time-varying gain. It is proved that the mean square containment problem can still be solved by the modified protocol if all agent-dependent gains are infinitesimals of the same order as time goes to infinity.

The notations used in this paper are: $1_n = (1, \ldots, 1) \in \mathbb{R}^n$; $0_n = (0, \ldots, 0) \in \mathbb{R}^n$; I_n denotes the $n \times n$ dimensional identity matrix; $\Theta_{m\times n} \in \mathbb{R}^{n\times m}$ denotes the $m \times n$ dimensional zero matrix (Θ_n denotes the $n \times n$ dimensional zero matrix); $\otimes$ denotes the Kronecker product and $\mathbb{Z}^+$ denotes the set of positive integers. For a given matrix X, $\|X\|_2$ denotes its 2-norm. diag$(\cdot)$ denotes a block diagonal matrix formed by its inputs. For the random variable x, $E(x)$ denotes its mathematical expectation. For a complex number c, $\Re(c)$ denotes its real part. For a set $S = \{x_1, \ldots, x_n\}$, the convex hull spanned by S is denoted by $\mathrm{co}(S) = \{\sum_{i=1}^{n} \alpha_i x_i | \alpha_i \in \mathbb{R}, \alpha_i \geq 0, \sum_{i=1}^{n} \alpha_i = 1\}$. For $r \in \mathbb{Z}^+$ and $\lambda \in \mathbb{R}$, $J_r(\lambda) \in \mathbb{R}^{r\times r}$ is a Jordan block with diagonal element being λ.

2. Preliminaries

In this paper, a multi-agent system composed of n agents is investigated. In the literature, the communication topology of MASs is usually modeled by a digraph $\mathcal{G} = \{\mathcal{V}_{\mathcal{G}}, \mathcal{E}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}\}$, where $\mathcal{V}_{\mathcal{G}} = \{v_1, v_2, \ldots, v_n\}$, $\mathcal{E}_{\mathcal{G}} \subseteq \mathcal{V}_{\mathcal{G}} \times \mathcal{V}_{\mathcal{G}} = \{e_{ij} | i, j = 1, 2, \ldots, n\}$ and $\mathcal{A}_{\mathcal{G}} = [\alpha_{ij}]_{n\times n}$ are the node set, edge set and adjacency matrix, respectively. Agent i is denoted by node v_i. There is an information flow from agent j to agent i if and only if there is a directed edge from node j to node i, namely $e_{ij} \in \mathcal{E}_{\mathcal{G}}$. Node j is called the parent node of node i if $e_{ij} \in \mathcal{E}_{\mathcal{G}}$. It is assumed that $e_{ij} \in \mathcal{E}_{\mathcal{G}} \Leftrightarrow \alpha_{ij} > 0$ and $e_{ij} \notin \mathcal{E}_{\mathcal{G}} \Leftrightarrow \alpha_{ij} = 0$. The neighborhood of v_i is defined by $\mathcal{N}_i = \{v_j | e_{ij} \in \mathcal{E}_{\mathcal{G}}\}$. $\deg_{in}(v_i) = \sum_j^n (\alpha_{ij})$ is called the *in-degree* of agent i. The Laplacian matrix of $\mathcal{G}$ is defined by $L_{\mathcal{G}} = \mathcal{D}_{\mathcal{G}} - \mathcal{A}_{\mathcal{G}}$, where $\mathcal{D}_{\mathcal{G}} = \mathrm{diag}(\deg_{in}(v_1), \deg_{in}(v_2), \ldots, \deg_{in}(v_n))$.

A directed path from node v_{i_0} to node v_{i_s} is a sequence of distinct nodes $v_{i_0}, v_{i_1}, \ldots, v_{i_s}$ such that $e_{i_{j+1}i_j} \in \mathcal{E}_{\mathcal{G}}, j = 0, \ldots, s-1$. A subgraph of $\mathcal{G}$ is a graph whose node set is a subset of $\mathcal{V}_{\mathcal{G}}$, and whose edge set is a subset $\mathcal{E}_{\mathcal{G}}$. And a subgraph of $\mathcal{G}$ is called a spanning subgraph of $\mathcal{G}$ if its node set is the same as $\mathcal{V}_{\mathcal{G}}$. Moreover, a subgraph is called strongly connected if there exists a path between any two distinct nodes of this subgraph. The maximal strongly connected subgraphs of $\mathcal{G}$ are called its strongly connected components. In the digraph $\mathcal{G}$, the node which has no parent node is called a root. A directed tree is a digraph which has a root, and whose every node, except the root, has exactly one parent node. A directed forest is a disjoint union of directed trees. A digraph $\mathcal{G}$ has a spanning tree/forest if there exists a directed tree/forest which is the spanning subgraph of $\mathcal{G}$ (Brualdi & Ryser, 1991).

In this paper, we adopt the definitions of the leader and the follower used in Cao and Ren (2009). That is: an agent is called a leader if and only if it has no neighbor, otherwise it is called a follower. It is assumed that agents $1, \ldots, m$ $(m < n)$ are leaders and the other agents $m+1, \ldots, n$ are followers. Therefore, the Laplacian matrix has the following structure:

$$L_{\mathcal{G}} = \begin{bmatrix} \Theta_{m\times m} & \Theta_{m\times(n-m)} \\ L_1 & L_2 \end{bmatrix}, \tag{1}$$

where $L_1 \in \mathbb{R}^{(n-m)\times m}$ and $L_2 \in \mathbb{R}^{(n-m)\times(n-m)}$.

Lemma 1 (*Li, Ren, Liu, & Fu, 2013; Meng, Ren, & You, 2010*)**.** *All the eigenvalues of L_2 defined in (1) have positive real parts if the digraph $\mathcal{G}$ has a spanning forest whose roots are exactly the leaders of MASs.*

Agent i is described by the following continuous-time first-order integral dynamics

$$\dot{x}_i = u_i(t), \tag{2}$$

where $x_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ represent the position and control input of agent i, respectively. Here, $x_i(t)$ and $u_i(t)$ are assumed to be one dimensional for the purpose of simplicity. Actually, it is easy to extend them to the high-dimensional case.

Agent exchanges its position with each other in a noisy communication environment. It is assumed that the information that agent i receives from its neighbor agent j is corrupted by the additive communication noise $\delta_{ij}\eta_{ij}(t)$, where $\eta_{ij}(t)$ is the standard white noise and $\delta_{ij} > 0$ is the finite noise intensity. Then the real information agent i receives from agent j is $\zeta_{ij}(t) = x_i(t) + \delta_{ij}\eta_{ij}(t)$. It is also assumed that $\{\eta_{ij}(t) | i = 1, \ldots, n; j = 1, \ldots, n\}$ are mutually independent. That is: $\forall \{i, j\} \neq \{k, l\}, E\{\eta_{ij}(t)\eta_{kl}(t)\} = 0$.

The control objective to solve the following mean square containment problem of MASs with communication noises.

Definition 1. The mean square containment problem of MASs with communication noises is said to be solved by $\{u_1(t), \ldots, u_n(t)\}$ in a distributed manner if for any initial states $(x_1(0), \ldots, x_n(0))^T \in \mathbb{R}^n$, there exist deterministic variables $x_i^* \in \mathrm{co}_L \triangleq \mathrm{co}\{x_{1f}, \ldots, x_{mf}\} \triangleq \{x_f | x_f = \sum_{i=1}^{m} k_i x_{if}, k_i \geq 0, \sum_{i=1}^{m} k_i = 1\}$ (co_L denotes the convex hull spanned by the leaders' final positions $x_{jf} \triangleq \lim_{t\to\infty} x_j(t), j = 1, \ldots, m$) such that $\lim_{t\to\infty} E|x_i(t) - x_i^*|^2 = 0$, $i = m+1, m+2, \ldots, n$. Moreover, the control input $u_i(t)$ of agent i is only dependent on the information of its neighbor agents $\mathcal{N}_i$.