Automatica 50 (2014) 1929–1934

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Output controllability and optimal output control of state-dependent switched Boolean control networks ☆

Hao Chen, Jitao Sun [1]

Department of Mathematics, Tongji University, Shanghai, 200092, PR China

ARTICLE INFO

Article history:
Received 10 March 2013
Received in revised form
13 February 2014
Accepted 4 April 2014
Available online 2 June 2014

Keywords:
Output controllability
State-dependent switch
Boolean networks
Semi-tensor product
Optimal control

ABSTRACT

In the present paper, we investigate the output-controllability and optimal output control problems of a state-dependent switched Boolean control network. By using the semi-tensor product, the algebraic form of the system is obtained. Then, output-controllability problems of the system are discussed and some necessary and sufficient conditions are given. Next, the Mayer-type optimal output control issue is considered and an algorithm is provided to find out the control sequence. At last, an example is given to show the effectiveness of the main results.

1. Introduction

Since the Boolean networks were proposed by Kauffman (1969) to model and quantitatively describe the gene regulatory, great attention has been attracted to the study of them from researchers in many fields, such as biology, system science, and so on; see Akutsu, Hayashida, Ching, and Ng (2007), Drossel, Mihaljev, and Greil (2005) and Liang, Fuhrman, and Somogyi (1998) for example. In Boolean networks, the state of a gene is described as active (1) or inactive (0) and the interactions between each gene are determined by Boolean functions. The researchers have been in lack of tools until the semi-tensor product is proposed in Cheng, Qi, and Li (2011) by Cheng. Using the new tool, numerous control problems have been studied, such as the stability and stabilization (Cheng, Qi, Li, & Liu, 2011; Li & Sun, 2012c), the controllability and observation (Cheng & Qi, 2009; Li & Sun, 2011, 2012a,b; Zhao, Kim, & Filippone, 2013), the realization (Cheng, Li, & Qi, 2010), the optimal control problem (Laschov & Margaliot, 2011; Li & Sun, 2012a; Zhao, Li, & Cheng, 2011) and so on.

Switched systems play an important role in the study of control theory. Because of the universality of switched systems, great attention has been drawn to the study of them and some excellent results have been obtained; see Branicky (1998) and Geromel and Deaecto (2009) and references therein. In practice, the dynamics of biological systems are often governed by switching models (El-Farra, Gani, & Christofides, 2005). Switched dynamics can be triggered by inner and external causes. When the Boolean network is applied in modeling biological systems, the dynamics become the switched Boolean network. Some fundamental problems have been investigated on switched Boolean control networks. In Li and Wang (2012), the authors considered the reachability and controllability of switched Boolean control networks.

Output-controllability and optimal control issues are fundamental concepts in a control theory field. There has been a great lot of literature studying on both topics; see Huang, Li, Duan, and Starzyk (2012), Jin, Yang, and Che (2012) and Kobayashi, Imura, and Hiraishi (2009) and references therein. To the best of our knowledge, referring to the controllability of switched Boolean control networks, there exist no results except (Li & Wang, 2012). State-dependent switching is one important kind of the switching signal with both theoretical meanings and practical applications; see Liberzon (2003) for example. However, to the best of our knowledge, there exists no result studying the state-dependent switched Boolean control networks, which motivates the present research on the output-controllability and optimal output control of state-dependent switched Boolean control networks.

In the paper, we first present the state-dependent switching Boolean control networks and obtain the algebraic form by using direct methods. The state-dependent switched Boolean control

☆ This work is supported by the National Natural Science Foundation of China under Grant 61174039. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Bart De Schutter under the direction of Editor Ian R. Petersen.

E-mail address: sunjt@sh163.net (J. Sun).

[1] Tel.: +86 21 65983241 1307; fax: +86 21 65981985.

http://dx.doi.org/10.1016/j.automatica.2014.05.013
0005-1098/

network can also be first transformed into a non-switched logical network. And then we can get the algebraic form of the system. However, our method in the paper is more direct and easier to understand. After getting the algebraic form of the system, necessary and sufficient conditions are obtained for the output-controllability of the system and an optimal output control design algorithm for the Mayer-type optimal output problem is also given. We focus on the investigation of the system at a theoretical level. We believe that it is meaningful and it may offer help for researchers in other fields, such as biological systems, systems science and so on.

The rest of the paper is organized as follows. In Section 2, some preliminaries, including some basic concepts, notations and propositions, used in the paper are introduced. The main results are presented in Section 3. State-dependent switching Boolean control networks are introduced, and the algebraic form of the system is obtained. The definition for the output-controllability of the system is provided. And then, the necessary and sufficient condition is given. The Mayer-type optimal output control problem is also considered, and an optimal output control design algorithm is provided. Section 4 shows an example to illustrate the main results obtained in the paper. Lastly, conclusions are presented in Section 5.

2. Preliminaries

In this section, we introduce some necessary preliminaries on the semi-tensor product, the crucial tool in the present paper. The matrix product is assumed to be the semi-tensor product in the following discussion. Following is a review of basic concepts, notations and proposition in Cheng, Qi, Li et al. (2011).

Definition 2.1 (*Cheng, Qi, Li et al., 2011*). (1) Let X be a row vector of dimension np, and $Y = [y_1, y_2, \ldots, y_p]^T$ be a column vector of dimension p. Then we split X into p equal-size blocks as $X^1, \ldots, X^p$, which are $1 \times n$ rows. Define the semi-tensor product, denoted by $\ltimes$, as

$$\begin{cases} X \ltimes Y = \sum_{i=1}^{p} X^i y_i \in R^n, \\ Y^T \ltimes X^T = \sum_{i=1}^{p} y_i (X^i)^T \in R^n. \end{cases}$$

(2) Let $M \in \mathcal{M}_{m\times n}$ and $N \in \mathcal{M}_{p\times q}$. If n is a factor of p or p is a factor of n, then $C = M \ltimes N$ is called the semi-tensor product of M and N, where C consists of $m \times q$ blocks as $C = (C^{ij})$, and

$C^{ij} = M^i \ltimes N_j, \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, q,$

where $M^i = Row_i(M)$ denotes the ith row of the matrix M and $N_j = Col_j(N)$ denotes the jth column of the matrix N.

Remark 2.1. The semi-tensor product is a generalization of the conventional matrix product. The semi-tensor product of two matrices $M \in \mathcal{M}_{m\times n}$ and $N \in \mathcal{M}_{p\times q}$ becomes the conventional matrix product for $n = p$.

Next, notations used in the following paper are given.

(1) $\mathcal{D} := \{0, 1\}$, $\Delta_n := \{\delta_n^1, \ldots, \delta_n^n\}$, where δ_n^k denotes the kth column of the identity matrix I_n.
(2) Let $\mathcal{M}_{n\times s}$ denote the set of $n \times s$ matrices. Assume that a matrix $M = [\delta_n^{j_1} \delta_n^{j_2} \cdots \delta_n^{j_s}] \in \mathcal{M}_{n\times s}$, i.e., its columns, $Col(M) \subset \Delta_n$; then M is called a logical matrix. The set of $n \times m$ logical matrices is denoted by $\mathcal{L}_{n\times m}$.
(3) To use a matrix expression, we identify $1 \sim \delta_2^1$, $0 \sim \delta_2^2$. Using this transformation, a logical function $f : \mathcal{D}^k \to \mathcal{D}$ becomes a function $f : \Delta_2^k \to \Delta_2$.
(4) Consider a fundamental unary logical function, Negation, $\neg P$, and four fundamental binary logical functions, Disjunction, $P \vee Q$; Conjunction, $P \wedge Q$; Conditional, $P \to Q$; Biconditional, $P \leftrightarrow Q$. Their structure matrices are as follows:
$M_\neg = \delta_2[2, 1]$; $M_\vee = \delta_2[1, 1, 1, 2]$; $M_\wedge = \delta_2[1, 2, 2, 2]$;
$M_\to = \delta_2[1, 2, 1, 1]$; $M_\leftrightarrow = \delta_2[1, 2, 2, 1]$.
(5) Define a swap matrix $W_{[m,n]}$, which is an $mn \times mn$ matrix constructed in the following way: label its columns by $(11, 12, \ldots, 1n, \ldots, m1, m2, \ldots, mn)$ and its rows by $(11, 21, \ldots, m1, \ldots, 1n, 2n, \ldots, mn)$. Then its element in the position $((I, J), (i, j))$ is assigned as
$$w_{(I,J),(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1, & I = i \text{ and } J = j, \\ 0, & \text{otherwise.} \end{cases}$$
(6) Define a matrix M_r, called the power-reducing matrix, as $M_r = \delta_4[1\ 4]$. Furthermore, construct the group power-reducing matrix as follows:
$$\Phi_j = \prod_{i=1}^{j} I_{2^{i-1}} \otimes [(I_2 \otimes W_{[2,2^{j-i}]})M_r]. \tag{1}$$
Using the matrix expression, the following proposition could be obtained.

Proposition 2.1 (*Cheng, Qi, Li et al., 2011*). (1) *Let $f(x_1, x_2, \ldots, x_k)$ be a logical function; then there exists a unique 2×2^k matrix M_f, called the structure matrix, such that*
$f(x_1, x_2, \ldots, x_k) = M_f x,$
where $x = \ltimes_{i=1}^k x_i \in \Delta_{2^k}$, $M_f \in \mathcal{L}_{2\times 2^k}$.
(2) *Let $x \in R^t$ and A be a given matrix. Then $xA = (I_t \otimes A)x$, where $\otimes$ denotes the Kronecker product.*
(3) *Let $E_d = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$. Then for any two logical variables $X, Y \in \Delta_2$, $E_d XY = Y$ or $E_d W_{[2,2]} XY = X$, where $W_{[2,2]}$ is the swap matrix.*

3. Main results

Consider a state-dependent switched Boolean control network:

$$\begin{cases} x_1(t+1) = f_1^{\sigma(t)}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)), \\ x_2(t+1) = f_2^{\sigma(t)}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)), \\ \vdots \\ x_n(t+1) = f_n^{\sigma(t)}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)), \end{cases} \tag{2}$$
$$y_j(t) = h_j(x_1(t), \ldots, x_n(t)), \quad j = 1, 2, \ldots, p\ (p \le n),$$

where $x_i(t) \in \mathcal{D}$, $i = 1, 2, \ldots, n$ are Boolean variables, $u_i(t) \in \mathcal{D}$, $i = 1, 2, \ldots, m$ are control inputs and $f_i^{\sigma(t)} : \mathcal{D}^{m+n} \to \mathcal{D}$, $i = 1, 2, \ldots, n$, $h_j : \mathcal{D}^n \to \mathcal{D}$, $j = 1, 2, \ldots, p$ are logical functions. $\sigma : \mathbb{N} \to W = \{1, 2, \ldots, w\}$ is the state-dependent switching signal.

Define $x(t) = \ltimes_{i=1}^n x_i(t)$, $u(t) = \ltimes_{i=1}^m u_i(t)$, $y(t) = \ltimes_{j=1}^p y_j(t)$. Assume that the structure matrices for $f_i^{\sigma(t)}$, h_j is $M_i^{\sigma(t)}$, S_j, respectively. Then, system (2) can be expressed in componentwise algebraic form as

$$\begin{cases} x_1(t+1) = M_1^{\sigma(t)} x(t) u(t), \\ x_2(t+1) = M_2^{\sigma(t)} x(t) u(t), \\ \vdots \\ x_n(t+1) = M_n^{\sigma(t)} x(t) u(t), \end{cases} \tag{3}$$
$$y_j(t) = S_j x(t), \quad j = 1, 2, \ldots, p.$$

Multiplying both sides of Eqs. (3), one obtains

$$\begin{cases} x(t+1) = L_{\sigma(t)} x(t) u(t), \\ y(t) = Hx(t), \end{cases} \tag{4}$$

where $Col_j(L_{\sigma(t)}) = \ltimes_{i=1}^n Col_j(M_i^{\sigma(t)})$, $j = 1, 2, \ldots, 2^{m+n}$, and $Col_j(H) = \ltimes_{i=1}^p Col_j(S_i)$, $j = 1, 2, \ldots, 2^n$.