Automatica 48 (2012) 1650–1657

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Disturbance and input–output decoupling of singular systems ☆

Dimitris Vafiadis [1], Nicos Karcanias

Control Engineering Centre, City University, London EC1V 0HB, England, United Kingdom

ARTICLE INFO

Article history:
Received 29 November 2010
Received in revised form
26 November 2011
Accepted 18 December 2011
Available online 13 June 2012

Keywords:
Singular systems
Decoupling
Disturbance rejection
Matrix fraction description

ABSTRACT

The disturbance decoupling and the simultaneous disturbance and input–output decoupling problems for singular systems are considered in the context of the matrix fraction description (MFD) of the system. Solvability conditions are obtained in terms of the composite matrix of a column reduced MFD of the system, a characterisation of the fixed poles of both problems is given and it is shown that the remaining poles can be arbitrarily assigned.

1. Introduction

Disturbance decoupling is one of the most studied problems in control. For the class of state space systems with proper or strictly proper transfer functions numerous papers were published over the last decades and several aspects of disturbance decoupled systems have been investigated (see for example Basile and Marro (1969), Wonham (1985) where the problem was tackled by using geometric control theory, Dion, Commault and Montoya (1994) where a structural approach was followed for the solution of diagonal and disturbance decoupling of a state space system). In Malabre, Martinez-Garcia, and Del-Muro-Cuellar (1997) the problem of fixed poles of disturbance decoupling was solved by using the geometric approach. In Koussiouris and Tzierakis (1996) the disturbance decoupling problem with input–output decoupling was solved in the frequency domain. The fixed poles of the latter problem were also considered in Camart, Malabre, and Martinez-Garcia (2001). Disturbance decoupling for singular or implicit systems has attracted the attention of researchers and several papers have been published: Fletcher and Asaraai (1989) was the first paper to tackle the problem. Other papers where important work was done are Ailon (1993, 1992) where the problem is solved and the stabilizability of the closed loop system is studied by using arguments and tools based on the state space systems. In Banaszuk, Kociecki and Przyluski (1990) and Lebret (1991) the disturbance decoupling problem has been considered for the case of implicit systems.

The aim of the present paper is to provide a frequency domain approach to the disturbance and input–output decoupling for singular systems. The approach used follows along the lines of Vafiadis and Karcanias (2003) where the block decoupling problem was considered. Although block decoupling and disturbance decoupling are different design goals, they have a major similarity when they are defined in the context of matrix fraction description (MFD) of the transfer function of the system: in both problems the desired resulting system has the property that certain rows of the numerator matrix lie in the rational vector space spanned by certain rows of the denominator matrix. This similarity naturally leads to similar methodologies for the solution of the above problems, individually and in combination. Frequency domain approach allows the use of common tools for problems of different nature. This is an advantage, comparing to dominant state space approaches used.

The MFD representations of nonproper systems have the characteristic that some of the pivot indices (Forney, 1975; Kailath, 1980) of a column reduced composite matrix of the MFD appear in the numerator matrix in contrast to the class of strictly proper (state space) systems where all pivot indices appear in the denominator matrix. In this way we have a classification of the pivot indices into proper and nonproper (Vafiadis & Karcanias, 1997b). When nonproper pivot indices exist (i.e. the transfer function of the system is non proper), state feedback can alter the "denominator matrix" of the system such that its column (row, in the case of left MFD) high order coefficient matrix can change. This is a consequence of the fact that for the case of singular systems feedback can change the structure at infinity.

☆ The material in this paper was presented at the 4th IFAC Symposium on System, Structure and Control (SSSC 2010), September 15–17, 2010, Ancona, Italy. This paper was recommended for publication in revised form by Associate Editor Fen Wu, under the direction of Editor Roberto Tempo.

E-mail addresses: dvaf12@gmail.com, dvaf@gsis.gr (D. Vafiadis), n.karcanias@city.ac.uk (N. Karcanias).

[1] Tel.: +44 20 7040 8125; fax: +44 20 7040 8568.

0005-1098/$ – see front matter
doi:10.1016/j.automatica.2012.05.021

The treatment of the problem and the methodology followed in the present paper is based on the above property of singular systems and the existence of non-proper controllability indices (Karcanias & Eliopoulou, 1990; Malabre, Kucera, & Zagalak, 1990) when the system is singular. Necessary and sufficient conditions are obtained for the existence of a solution to the disturbance decoupling and simultaneous disturbance and input–output decoupling problems. The conditions are easily testable and can be derived from the MFD of the disturbed system. The proof of the sufficiency of the solvability conditions provides a constructive way for selecting the feedback matrices solving the problem.

For both disturbance decoupling and simultaneous disturbance and input–output decoupling problems, the set of fixed poles is characterised in terms of the MFD of the system in a way analogous to that of state space systems (see Koussiouris and Tzierakis (1996) and Camart et al. (2001)).

In what follows the disturbance decoupling problem for singular systems will be referred to as DDSS, while the combined disturbance and input–output decoupling as DDDSS. The following notation will be used: the row (column) high order coefficient matrix (Kailath, 1980) of a polynomial matrix $P(s)$ will be denoted by $[P]_{hr}$ ($[P]_{hc}$). The row span over $\mathbb{R}$ ($\mathbb{R}(s)$) of a matrix P will be denoted by $span_{\mathbb{R}}\{P\}$ ($span_{\mathbb{R}(s)}\{P\}$). The notation $(N(s), D(s))$ will be used when we refer to a system with composite matrix $T(s) = [N^T(s), D^T(s)]^T$. A singular system with matrices E, A, B, C will be denoted by (E, A, B, C) and the feedback law $u = Fx + Gv$ will be referred to as feedback pair (F, G). The matrix $[T]_{hc}$ will be written as $[N_{hc}^T, D_{hc}^T]^T$, $N_{hc} \in \mathbb{R}^{m\times\ell}$, $D_{hc} \in \mathbb{R}^{\ell\times\ell}$.

2. DDSS problem statement and preliminaries

Consider the singular system

$$E\dot{x} = Ax + Bu + \Xi\xi, \qquad y = Cx, \qquad det(sE - A) \neq 0 \tag{1}$$

where $E \in \mathbb{R}^{n\times n}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times\ell}$, $C \in \mathbb{R}^{m\times n}$ and $\Xi \in \mathbb{R}^{n\times d}$ i.e. the system has n states, ℓ inputs, m outputs and d disturbance inputs. Matrix E may be singular. Our goal is the determination of the necessary and sufficient conditions for the elimination of the influence of the disturbance $\xi(t)$ on the output $y(t)$ by means of state feedback of the type

$$\begin{aligned} &u = Fx + Gv, \quad F \in \mathbb{R}^{\ell\times n},\ G \in \mathbb{R}^{\ell\times\ell} \\ &det(G) \neq 0, \qquad det(sE - A - BF) \neq 0 \end{aligned} \tag{2}$$

and the method of construction of such feedback laws. Throughout the paper it will be assumed that the system (1) is reachable, i.e. $[sE-A, B, \Xi]$ has no finite zeros and $[E, B, \Xi]$ has full row rank. It will also be assumed that B is monic and $m \leq \ell$. From (1) and (2) it follows

$$\begin{bmatrix} u \\ \xi \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} x + \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} v \\ \xi \end{bmatrix} \tag{3}$$

or

$$\begin{bmatrix} u \\ \xi \end{bmatrix} = \hat{F}x + \hat{G}\begin{bmatrix} v \\ \xi \end{bmatrix}. \tag{4}$$

Notice that $\hat{G}$ is invertible as long as G is invertible. Let

$$H(s) = N(s)D^{-1}(s) \tag{5}$$

be a coprime and column reduced MFD of the transfer function of (1). Then we have (see Vafiadis and Karcanias (1997b) and Vafiadis and Karcanias (2003)) that the closed loop MFD has "numerator" and "denominator"

$$N(s) = CS(s), \qquad D_c(s) = \hat{G}^{-1}[D(s) - \hat{F}S(s)] \tag{6}$$

where $S(s) = \operatorname{diag}\{[1, s, \ldots, s^{r_i-1}]^T\}$, with $r_i,\ i = 1, \ldots, \ell + d$ being the reachability indices of $(E, A, [B\ \Xi])$. Then from (3)

$$D_c(s) = \begin{bmatrix} G^{-1} & 0 \\ 0 & I \end{bmatrix} \left[D(s) - \begin{bmatrix} F \\ 0 \end{bmatrix} S(s) \right]. \tag{7}$$

Now partitioning $D(s)$ conformably to the block partitioning of $\hat{G}$, $\hat{F}$ i.e. if

$$D(s) = \begin{bmatrix} D_u(s) \\ D_\xi(s) \end{bmatrix}, \qquad D_u(s) \in \mathbb{R}^{\ell\times(\ell+d)}[s] \quad D_\xi(s) \in \mathbb{R}^{d\times(\ell+d)}[s] \tag{8}$$

it follows from (7) that

$$D_c(s) = \begin{bmatrix} G^{-1}[D_u(s) - FS(s)] \\ D_\xi(s) \end{bmatrix} \triangleq \begin{bmatrix} D_v(s) \\ D_\xi(s) \end{bmatrix}. \tag{9}$$

The meaning of the above is that state feedback of type (2) on system (1) affects only the top ℓ rows of the closed loop system denominator matrix.

Definition 1. The row span (over $\mathbb{R}$) of the row high order coefficient matrix $[P]_{hr}$ of a row reduced polynomial matrix (Kailath, 1980) $P(s)$ is referred to as the *highest degree characteristic space* of the rational vector space spanned by the rows of $P(s)$ and is denoted by $\mathscr{L}\{P(s)\}$. □

Some useful properties of the highest degree characteristic space are given below.

Lemma 2 (*Karcanias, 1996, 1994; Koussiouris, 1979*). (i) *All row reduced bases of a rational vector space have the same highest degree characteristic space.* (ii) *If $[P]_{hr}$ is the high order coefficient matrix of $P(s)$ then $span_{\mathbb{R}}\{[P]_{hr}\} \subseteq \mathscr{L}\{P(s)\}$.* (iii) *If $P_1(s)$ and $P_2(s)$ are two polynomial matrices such that $span_{\mathbb{R}(s)}\{P_1(s)\} \subseteq span_{\mathbb{R}(s)}\{P_2(s)\}$ then $\mathscr{L}\{P_1(s)\} \subseteq \mathscr{L}\{P_2(s)\}$.* □

The pivot indices (p.i.) of a column reduced basis of a rational vector space play an important role in the paper. Their definition is the following.

Definition 3 (*Forney, 1975*). Let V be a column reduced basis of a vector space over $\mathbb{R}(s)$ with ordered column degrees $\nu_1 \leq \cdots \leq \nu_\ell$.

The pivot indices $q_1 \cdots q_\ell$ are defined as follows: let V have n_1 columns with degree ν_1. Find the first (lowest index) n_1 rows of V_{hc} such that the $n_1 \times n_1$ submatrix of V_{hc} so defined is nonsingular. The indices of these rows, in order, form the first n_1 pivot indices $q_1 \cdots q_{n_1}$. Delete these n_1 columns and n_1 rows from V and repeat the above procedure to find the next group of pivot indices, corresponding to the columns with the next distinct index value; and so forth. □

Pivot indices of $T(s) = [N^T(s), D^T(s)]^T$ can be classified into two types (Vafiadis & Karcanias, 1997b).

Definition 4. Let $q_1, \ldots, q_{\ell+d}$ denote the pivot indices of $[N^T(s), D^T(s)]^T$. Then q_i is called **proper** if $q_i > m$ and **nonproper** if $q_i \leq m$. □

The entries (q_i, i) of $T(s)$ will be referred to as *pivot elements* and are classified into proper and nonproper pivot elements according to the above definition. Furthermore the rows of $T(s)$ that contain pivot elements will be referred to as *pivot rows*. The matrix $T(s)$ is a basis of the vector space spanned by its columns. Throughout the rest of the paper it will be assumed that the given system has τ nonproper p.i.

Definition 5. The integers $p_i, \overline{p}_i, \tau$ are defined as follows:

(i) p_i are the column indices of $T(s)$ such that the corresponding p.i. q_{p_i} are proper.