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Brief paper

Adaptive consensus of multi-agents in networks with jointly connected topologies[☆]

Hui Yu [a,b,1], Xiaohua Xia [b]

[a] College of Science, China Three Gorges University, Yichang 443002, China

[b] Centre of New Energy Systems, Department of Electrical, Electronics and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa

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ABSTRACT

In this paper, the consensus problem of multi-agent following a leader is studied. An adaptive design method is presented for multi-agent systems with non-identical unknown nonlinear dynamics, and for a leader to be followed that is also nonlinear and unknown. By parameterizations of unknown nonlinear dynamics of all agents, a decentralized adaptive consensus algorithm is proposed in networks with jointly connected topologies by incorporating local consensus errors in addition to relative position feedback. Analysis of stability and parameter convergence of the proposed algorithm are conducted based on algebraic graph theory and Lyapunov theory. Finally, examples are provided to validate the theoretical results.

1. Introduction

Distributed coordination of a group of dynamical agents is of interest in control and robotics. This is due to the broad applications of multi-agent systems in many areas, e.g., in multi-vehicle rendezvous, formation control of multi-robots, flocking, swarming, distributed sensor fusion, attitude alignment, and congestion control in communication networks. An important problem in distributed coordinated networks of dynamical agents is to find a distributed control law so that all agents can reach consensus on a common decision value. This problem is the so-called consensus problem.

Early well-known works on consensus coordination for networks of dynamical agents have been done in the context of control theory in Fax and Murray (2004), Hatano and Mesbahi (2005), Jadbabaie, Lin, and Morse (2003), Lin, Broucke, and Francis (2004), Moreau (2005), Olfati-Saber and Murray (2004), Ren and Beard (2005), Savkin (2004), to name just a few. In recent years, relevant topics on consensus problem have been extensively further investigated in different situations, for example, consensus in networks with time-delays (Sun & Wang, 2009; Zhu & Cheng, 2010), finite-time consensus (Khoo, Xie, & Man, 2009), consensus in stochastic networks (Tahbaz-Salehi & Jadbabaie, 2008), quantized consensus (Kashyap, Basar, & Srikant, 2007), etc.

Recently, an interesting topic is the consensus problem of a group of agents with unknown information. In Hong, Hu, and Gao (2006), the authors proposed a consensus algorithm of agents with an active leader with unmeasurable state and variable interactive topology. The algorithm is also extended to the case that the interconnected graphs of agents are not always connected in intervals with identical length. In Bai, Arcak, and Wen (2008, 2009), the authors studied a coordination problem steering a group of agents to a formation that translates with a prescribed reference velocity. Decentralized adaptive designs are proposed for reference velocity recovery using relative position feedback in Bai et al. (2008) and tracking of the reference velocity by incorporating relative velocity feedback in addition to relative position feedback in Bai et al. (2009). In Hou, Cheng, and Tan (2009), the authors proposed a robust decentralized adaptive control approach using neural network to solve consensus problem of multi-agents with uncertainties and external disturbances in undirected networks. In Das and Lewis (2010), the authors presented a design method for adaptive synchronization controllers for distributed systems having non-identical unknown nonlinear dynamics, and for a

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E-mail addresses: yuhui@ctgu.edu.cn (H. Yu), xxia@postino.up.ac.za (X. Xia).

[1] Tel.: +86 717 6392618.

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target dynamics to be tracked that is also nonlinear and unknown. Under some assumptions, the authors proved that the overall local cooperative error vector and the neural network weight estimation errors are both uniformly ultimately bounded. In Yu, Lü, Chen, Duan, and Zhou (2009), for an unknown regulatory network with time delay and uncertain noise disturbance, an adaptive filtering approach is proposed to ensure the stochastic stability of the error states between the unknown network and the estimated model. Other kinds of adaptive synchronization design of complex dynamical networks are by using adaptive tuning of the coupling strength (Yu, Chen, & Lü, 2009), network weights, etc.

In this paper, we consider the adaptive consensus coordination problem of a group of agents with non-identical unknown nonlinear dynamics in networks with jointly connected topologies following a leader with also unknown nonlinear velocity dynamics. By parameterizing the unknown nonlinear dynamics of all agents by some basis functions, each agent estimating the unknown parameters, a decentralized adaptive consensus algorithm is developed in networks with jointly connected topologies by using both relative position feedback and local consensus error feedback of neighboring agents. By introducing *Persistent excitation* (PE) condition for regressor matrix, both position errors and parameter estimate errors can be proved to be globally uniformly asymptotically convergent to zero based on the algebraic graph theory and Lyapunov theory.

The contributions of this paper are mainly in two aspects. First, a novel type decentralized adaptive consensus control scheme is proposed for the considered multi-agent systems to follow a leader in networks with jointly connected topologies, by relative position and local consensus error feedback. When unknown information or unmeasured information exists in the system, there are few efforts in the literature considering networks with switching topology, especially jointly connected topologies. Except for Hong et al. (2006), the works (Bai et al., 2008, 2009; Das & Lewis, 2010; Hou et al., 2009; Yu, Chen et al., 2009; Yu, Lüet al., 2009) mentioned above are all for networks with fixed topology. In Hong et al. (2006), the case of networks with switching topologies and an extended case are studied. However, the algorithm proposed in Hong et al. (2006) is not strictly decentralized because each agent in the group must have access to the information $a_0(t)$ of the leader. Moreover, in the extended case of networks with switching topologies, it requires that each time interval has identical length, and the total period of connected interconnected graphs is sufficiently large. In our case, only jointly connectedness is assumed. Second, sufficient conditions are obtained for ensuring consensus with global, uniform and asymptotical parameter convergence. The consensus of all agents is ensured due to joint connectedness of graphs in networks with jointly connected topologies. The PE condition and some boundedness assumptions are introduced for ensuring parameter convergence. The parameter convergence analysis is more challenging when the interaction topology is switching. This is particularly true for the case of networks with jointly connected topologies, because standard results from adaptive control theory cannot be applied to the system directly. The two papers (Bai et al., 2008, 2009) also introduced PE condition for parameter convergence analysis in fixed network topology. A situation of all followers and the leader having non-identical unknown nonlinear dynamics and external disturbances is considered in Das and Lewis (2010), in which all consensus errors and parameter estimate errors are proved to be uniformly ultimately bounded (UUB) based on some assumptions in fixed network topologies. In Yu, Lüet al. (2009), parameter convergence is not considered. In our work, both consensus errors and parameter estimate errors converging to zero (globally uniformly asymptotically) are obtained for switching networks with joint connectedness.

This paper is organized as follows. In Section 2, we establish the notation and formally state the problem. We present our main results in Section 3, the simulation results supporting the objectives of the paper in Section 4 and the concluding remarks in Section 5.

2. Problem statement

We consider a multi-agent system consisting of N agents and a leader. The dynamics of N agents are described by

$$\dot{x}_i(t) = f_i(x_i, t) + u_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) \in R$ is the position state of ith agent, $u_i(t) \in R$ is the control input, and $f_i(x_i, t)$ is the dynamics of agent i, which is assumed to be unknown. Standard assumptions for existence of unique solutions are made, i.e., $f_i(x_i, t)$ is continuous in t and Lipschitz in x_i. We assume that the leader–agent moves in R and its underlying dynamics is described by

$$\dot{x}_0(t) = v_0(t) \tag{2}$$

where $x_0(t) \in R$ is the position state of the leader, $v_0(t) \in R$ is its velocity and assumed to be unknown. The leader–agent moves freely or along some planning trajectory, however, we assume that its velocity dynamics $v_0(t)$ is only related to time t and unknown.

Remark 1. For avoiding complicated expressions, the states of all agents are assumed to be scalars in R, which is trivial to be extended to R^n by introducing the Kronecker product. The Kronecker product of matrix $A \in R^{m\times n}$ and $B \in R^{p\times q}$ is defined as

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}.$$

The information exchange between agents in a multi-agent system can be modeled using graphs. A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a node set $\mathcal{V} = \{1, 2, \ldots, N\}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, where an edge of the edge set $\mathcal{E}$ is denoted by (i, j). A graph is undirected if edges $(i, j) \in \mathcal{E}$ are an unordered pair. A graph is simple if it has no self-loops or repeated edges. If there is an edge between two nodes, then the two nodes are neighbors (or adjacent) to each other. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}, j \neq i\}$. A path is a sequence of connected edges in a graph. If there is a path between any two nodes of a graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected, otherwise disconnected. The union of a collection of graphs is a graph with a node set and an edge set being the union of the node set and the edge set of all of the graphs in the collection. We say that a collection of graphs is jointly connected if the union of its members is connected.

With regarding the N agents as the nodes in $\mathcal{V}$, the relationships between N agents can be conveniently described by a simple and undirected graph $\mathcal{G}$, in which an undirected edge (i, j) denotes that agent i and j can sense, receive or obtain information from each other. The adjacency matrix of graph $\mathcal{G}$ is denoted by $A = [a_{ij}] \in R^{N\times N}$, whose (ij)th entry is 1 if (i, j) is an edge of graph $\mathcal{G}$ and 0 if it is not. The degree matrix $D \in R^{N\times N}$ of graph $\mathcal{G}$ is a diagonal matrix with ith diagonal element being $|\mathcal{N}_i|$. The Laplacian of graph $\mathcal{G}$ is defined as $L = D - A$, which is symmetric and have the following well-known results in algebraic graph theory (Godsil & Royle, 2001).

Lemma 2. *Laplacian L of graph $\mathcal{G}$ has at least one zero eigenvalue with $\mathbf{1}_N = (1, 1, \ldots, 1)^T \in R^N$ as its eigenvector, and all the non-zero eigenvalues of L are positive. Laplacian L has a simple zero eigenvalue if and only if graph $\mathcal{G}$ is connected.*