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Brief paper

Robust stability of packetized predictive control of nonlinear systems with disturbances and Markovian packet losses[☆]

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ABSTRACT

We study a predictive control formulation for uncertain discrete-time non-linear uniformly continuous plant models where controller output data is transmitted over an unreliable communication channel. The channel introduces Markovian data-loss and does not provide acknowledgments of receipt. To achieve robustness with respect to dropouts, at every sampling instant the controller transmits packets of data. These contain possible control inputs for a finite number of future time instants, and minimize a finite horizon cost function. At the actuator side, received packets are buffered, providing the plant inputs. Within this context, we adopt a stochastic Lyapunov function approach to establish stability results of the networked control system. A distinguishing aspect of this work is that it considers situations where the maximum number of consecutive packet dropouts has unbounded support.

1. Introduction

Motivated by both practical and also theoretical aspects, significant research has concentrated on Networked Control Systems (NCSs), as documented, e.g., in Baillieul and Antsaklis (2007). In an NCS, plant and controller communicate via a network which may be shared with other applications. This simplifies the cabling (especially if the network is wireless) and, thus, increases overall system reliability. However, since general purpose network platforms were not originally designed for applications with critical timing requirements, their use for closed-loop control presents some serious challenges. The network itself is a dynamical system that exhibits characteristics which traditionally have not been taken into account in control system design. In addition to being quantized, transmitted data may be affected by time delays and data-dropouts. Thus, in an NCS links are not transparent, often constituting a significant performance bottleneck. Various models have been utilized to describe time-delays and packet dropouts in NCSs. For example, suitable deterministic boundedness assumptions can be used to derive sufficient conditions for deterministic stability (Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2009; Hassibi, Boyd, & How, 1999; Muñoz de la Peña & Christofides, 2008a; Nešić & Teel, 2004; Tabbara, Nešić, & Teel, 2007; Walsh, Ye, & Bushnell, 2002; Xiong & Lam, 2009). A simple stochastic approach considers network effects as independent and identically distributed (i.i.d.) random variables (Antunes, Hespanha, & Silvestre, 2009, 2010; Bernardini, Donkers, Bemporad, & Heemels, 2010; Chatterjee, Amin, Hokayem, Lygeros, & Sastry, 2010; Gao, Meng, & Chen, 2008; Imer, Yüksel, & Başar, 2006; Ishii, 2009; Liang, Chen, & Pan, 2010; Matveev & Savkin, 2005; Schenato, Sinopoli, Franceschetti, Poolla, & Sastry, 2007; Shi, Xie, & Murray, 2009; Tabbara & Nešić, 2008; Tsumura, Ishii, & Hoshina, 2009; Zhao, Gao, & Chen, 2009). However, fading communication channel gains and network congestion levels are, in general, correlated (Elliot, 1963; Gilbert, 1960; Lindbom, Ahlén, Sternad, & Falkenström, 2002). This motivates the adoption of, more general, (finite) Markov chain models (Huang & Dey, 2007; Seiler & Sengupta, 2005; Shi & Yu, 2009; Smith & Seiler, 2003; Wu & Chen, 2007; Xie & Xie, 2009; Xiong & Lam, 2007; You & Xie, 2011).

An important feature of many communication protocols is that data is sent in large time-stamped packets. This opens the possibility to conceive NCS architectures in which *packets of data* containing finite sequences are sent through the network. Through buffering and appropriate selection logic at the receiver node, time

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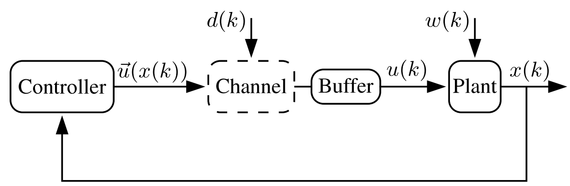

Fig. 1. NCS architecture with packet dropouts and buffering.

delays and packet dropouts can to some extent be compensated for Hu, Liu, and Rees (2007), Montestruque and Antsaklis (2004) and Quevedo, Silva, and Goodwin (2008). Here, model predictive control (MPC) (Rawlings & Mayne, 2009) becomes a natural choice for tackling controller to actuator links, since potential plant input values over a finite horizon are readily available (Casavola, Mosca, & Papini, 2006). Deterministic stability results of such *packetized predictive control* (PPC) schemes have been obtained in Ishido, Takaba, and Quevedo (2011), Muñoz de la Peña and Christofides (2008b), Quevedo and Nešić (2011) for cases where the maximum number of consecutive packets dropouts is bounded, and in Pin and Parisini (2011) for networks with bounded time-delays. Despite the widespread use of stochastic models in the communications community, to date only our own papers (Quevedo & Nešić, 2010; Quevedo, Østergaard, & Nešić, 2011) have studied stochastic stability of PPC. Whilst (Quevedo et al., 2011) focuses on quantized control of perturbed LTI systems with i.i.d. dropouts, Quevedo and Nešić (2010) considers nonlinear systems without disturbances in the presence of i.i.d. dropouts.

In the present work we study a PPC formulation for discrete-time non-linear plant models with disturbances, where optimizing sequences are transmitted over an unreliable communication channel, see Fig. 1. The controller is designed without knowledge of the packet dropout distribution and does not require acknowledgments of receipt. We combine elements of the PPC model of Quevedo and Nešić (2011) with stochastic stability analysis (Kushner, 1971; Meyn, 1989) to establish sufficient conditions for the optimal MPC value function to constitute a stochastic Lyapunov function of the NCS *at the successful transmission instants*. We then show how this property ensures stochastic stability of the NCS. Our stability results apply to NCSs with Markovian packet dropouts and nonlinear plant models with disturbances. Disturbances and times between successful transmissions are allowed to have unbounded support.

Notation. We write $\mathbb{R}$ for the real numbers, $\mathbb{R}_{>0}$ for $(0, \infty)$, $\mathbb{N}$ for $\{1, 2, \dots\}$, and $\mathbb{N}_0$ for $\mathbb{N} \cup \{0\}$. The $p \times p$ identity matrix is denoted via I_p; $0_p \triangleq 0 \cdot I_p$; $\{y\}_{\mathbb{K}} = \{y(\ell) : \ell \in \mathbb{K}\}$, and

$$\{y\}_{\ell_1}^{\ell_2} = \begin{cases} \{y(\ell_1), \dots, y(\ell_2)\} & \text{if } \ell_1 \leq \ell_2, \\ \{\,\} & \text{if } \ell_1 > \ell_2. \end{cases}$$

We adopt the convention $\sum_{k=\ell_1}^{\ell_2} a_k = 0$, if $\ell_1 > \ell_2$ and irrespective of a_k. The norm of a vector x is denoted $|x|$. To denote the probability of an event Ω, we write $\mathbf{Pr}\{\Omega\}$. The conditional probability of Ω given Γ is denoted via $\mathbf{Pr}\{\Omega \mid \Gamma\}$. The expected value of a random variable ν given Γ, is denoted by $\mathbf{E}\{\nu \mid \Gamma\}$, whereas for the unconditional expectation we will write $\mathbf{E}\{\nu\}$. We use the same notation for random variables and their realizations.

2. NCS architecture

We consider (possibly unstable) systems with state $x \in \mathbb{R}^n$ and constrained input $u \in \mathbb{U} \subseteq \mathbb{R}^p$, $0 \in \mathbb{U}$, described via:

$$x(k+1) = f(x(k), u(k), w(k)), \quad k \in \mathbb{N}_0, \tag{1}$$

where $f(0, 0, 0) = 0$. The initial state $x(0)$ is arbitrarily distributed (with possibly unbounded support) and the disturbance $\{w\}_{\mathbb{N}_0}$ is i.i.d., but otherwise arbitrarily distributed.[2]

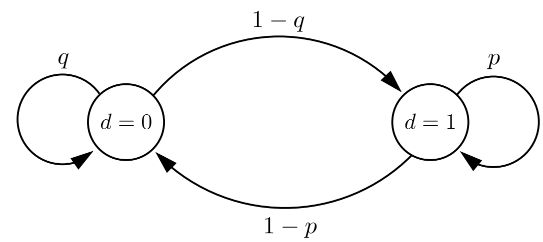

Fig. 2. Markov packet dropout model.

Network effects. Our interest lies in clock-driven networks situated between controller output and plant input. All data to be transmitted is sent in large time-stamped packets. The network is affected by transmission errors (for example, due to channel fading and congestion), which are in general correlated in time and introduce packet-dropouts (Elliot, 1963; Gilbert, 1960; Lindbom et al., 2002). This motivates us to model the network as an erasure channel and to characterize transmission effects via the following time-homogeneous binary Markov process $\{d\}_{\mathbb{N}_0}$:

$$d(k) \triangleq \begin{cases} 1 & \text{if packet-dropout occurs at instant } k, \\ 0 & \text{if packet-dropout does not occur at instant } k, \end{cases} \tag{2}$$

where the transition probabilities are given by, see Fig. 2:

$$\begin{aligned} &\mathbf{Pr}\{d(k+1) = 0 \mid d(k) = 0\} = q \\ &\mathbf{Pr}\{d(k+1) = 1 \mid d(k) = 0\} = 1 - q \\ &\mathbf{Pr}\{d(k+1) = 1 \mid d(k) = 1\} = p \\ &\mathbf{Pr}\{d(k+1) = 0 \mid d(k) = 1\} = 1 - p. \end{aligned} \tag{3}$$

The associated *failure rate* is $1 - q \in (0, 1)$, whereas the *recovery rate* is given by $1 - p \in (0, 1)$. The values $q \approx 1$ and $p \approx 0$, thus, describe a more reliable network; $q \approx 0$ and $p \approx 1$ refer to a network more prone to dropouts. The model (3) incorporates temporal correlations of network conditions. It is therefore more general and realistic than i.i.d. Bernoulli models. In fact, the i.i.d. dropout model corresponds to the special case where $q = 1 - p$, in which case:

$$\mathbf{Pr}\{d(k) = 1\} = p, \qquad \mathbf{Pr}\{d(k) = 0\} = 1 - p, \tag{4}$$

where p is the *dropout-rate*. In practice, p and q are not known exactly. Accordingly, in the present work our focus is on situations where the controller does not have knowledge about p and q. (Of course, closed loop stability will depend upon these parameters, see Sections 5 and 6.)

As foreshadowed in the introduction, at each time instant k and for plant state $x(k)$, the packetized predictive controller sends a control packet, say $\vec{u}(x(k))$, to the plant input node. To achieve good performance despite unreliable communication, $\vec{u}(x(k))$ contains constrained tentative control inputs for a finite number of N future time instants, i.e., we have

$$\vec{u}(x(k)) = \begin{bmatrix} u_0(x(k)) \\ u_1(x(k)) \\ \vdots \\ u_{N-1}(x(k)) \end{bmatrix} \in \mathbb{U}^N \subseteq \mathbb{R}^{pN}. \tag{5}$$

At the actuator side, the received packets are buffered, providing the plant inputs, see Fig. 1.

Buffering. The buffering mechanism amounts to a parallel-in serial-out shift register, which acts as a safeguard against dropouts.

[2] Note that our disturbance model serves to describe a class of NCSs with quantized inputs (Quevedo et al., 2011). Our results in Section 5 require that $\mathbf{E}\{|x(0)|^s\}$ and $\mathbf{E}\{|w(k)|^s\}$ be bounded for some $s > 0$.